EPISTOLA
AD
AMICUM
DE
COTESII Inventis,
CURVARUM RATIONE,
QUÆ
Cum Circulo & Hyperbola
Comparationem Admittunt.

LONDINI:
Impensis Guil. & Joh. Innys. MDCCXXXIII.

C. 3701.

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MVSEVM BRITANNICVM

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PRÆFATIO.

CUM primum eam clarissimi Cotesii operis partem evolveram, quæ Harmonia Mensurarum vocatur, in qua curvas comparandi ratio cum conicis sectionibus traditur, argumentum illud a viro eruditissimo adeo eleganter tractatum fuisse, res mihi visu fuit jucundissima. Speciatim vero curvarum areas, quarum ordinatim applicatis binomii rationales divisores tribuuntur, circuli sectionibus exhibendi viam non potui non magnopere admirari. Veruntamen cum hæc sine demonstrationibus suis nude proponerentur, theorematis, quo ea nitebantur, probationem magno videndi desiderio tenebar. Amicum igitur rogavi, licet aliis rebus necessariis tunc occupatum, ut temporis nonnihil ex gravioribus negotiis ereptum in hoc argumento impenderet. Quapropter hanc ab eo epistolam, tanta est illius in amicis gratificandis facilitas, cito postea accepi; qua non modo omnia demonstravit & explicuit, quæ Cotesius invenit; sed etiam limites istos a nonnullis partibus operis sustulit, quod quo minus alter faceret, mors immatura prævenit. Epistola autem hæc, etsi raptim & festinanter scripta, inter legendum adeo me delectavit,

EPI-

lectavit, ut dignam omnino esse, quæ lucem aspiceret, judicarem. Primum igitur inter miscellanea doctorum opera alicubi edere statueram; sed postea, mutato proposito, latine versam in publicum seorsim proferre consultius videbatur. Opusculum hoc iis haud ingratum fore, qui his rebus delectantur, nequaquam dubito. Neque cum ita dico, judicio nostro id minus tribui velim, quam in optimum amicum affectui, cujus summa integritas cum tot aliis virtutibus conjuncta jucundissimæ inter nos necessitudini occasionem dedit, quæ jam per multos annos constanter utrinque culta fuit.

Londini,
Augusti. 1. 1722.

J. W.

EPISTOLA

DE

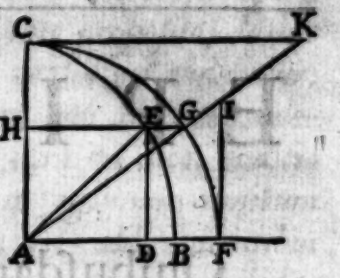
Quibuſdam COTESII Inventis.

Amico ſuo J. W. *integerrimo dilectiſſimoque* H. P. *ſalutem.*

CUM nuper petiiſti, vir chariſſime, ut ſenſus tibi meos de profectu aperiam, quem *Coteſius* in fluxionum doctrina fecit, quædam de eo ad te ſcribam: ubi conabor tum demonſtrare, quæ *Coteſius* invenit; tum pauca ſupplere, quæ a perſpicaci illo viro melius accepiſſemus, niſi mors ejus improviſa & lugubris impediiſſet. Ut primo *Coteſii* operis conſpectu admodum es lætatus, quod ſummi *Newtoni* doctrina fluxionum tam eleganter exculta fuit; non dubito, ſi antea hoc in mentem non occurrerit, quin multo majori voluptate ſis intellecturus omnia hæc *Coteſii* inventa ab iis deduci, quæ illuſtriſſimus hujus doctrinæ auctor prius ſcripſerat; unde facile intelligi poteſt, quo quis *Newtoni* eximium de curvarum quadratura librum diligentius excuſſerit, eo majorem ſpem concipiat hanc arduam matheſeos partem novis aliquibus acceſſionibus adornandi.

QUONIAM *Coteſii* inventa in duas partes dividuntur, quarum prior tabulas curvarum complectitur a ſe ipſo compoſitas; altera tabulas, quas ex chartis ejus doctiſſimus amicus tuus *Robertus Smith* magni *Coteſii* ſucceſſor digniſſimus conſtruxit; utraſque partes ordine tractabo. Ut a priori incipiamus, tabula ejus prima nihil differt a prima forma in tabula *Newtoni*, quæ curvarum menſuram exhibet a circulo & hyperbola pendentium. Secunda tabula cum ſecunda formarum *Newtoni* convenit, & ratio, qua tabulæ hujus curvæ metiuntur, a forma memorata *Newtoni* facile deducitur. Curvæ, cujus abſciſſa eſt z & ordinata $\frac{ez^{\frac{i}{n}-1}}{e+fz^{\frac{i}{n}}}$, area determinatur a *Newtono* modo ſequenti.

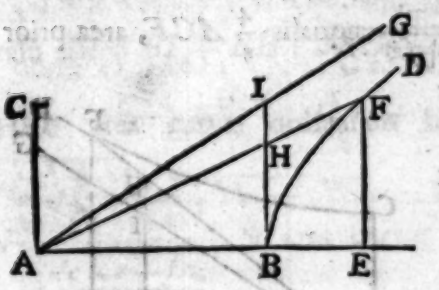
QUANDO area ad ellipſin referri poteſt, nimirum ſi ambo termini in denominatore $e+fz^n$ ſunt affirmativi, linea AB denotet unitatem, cui ad perpendiculum ducta ſit $AC=\sqrt{\frac{e}{f}}$; deinde ſemiaxibus AB, AC deſcribatur quarta pars ellipſeos; & ſumpta $AD=\sqrt{\frac{e}{e+fz^n}}$, ordinatim applicetur DE, & jungatur AE. Deinde area quæſita erit $=\frac{4}{n}ABE$. Jam ut *Coteſii* ratio hinc deducatur, deſcribendus eſt circuli quadrans CGF, & ducenda linea HEG ad AB parallela, linea quoque AG, item FI circulum contingens. Dein ut ſpat. ABE : ſpat. AFG $(=\frac{1}{2}AF\times FG)$ :: AB : AF; unde area ABE eſt $=\frac{1}{2}FG\times AB$. Rurſus ut HE : HG :: AB : AF, & ut HG : AG :: AF : AI; ex æquo igitur ut HE $(=\sqrt{\frac{e}{e+fz^n}})$: AG $(=\sqrt{\frac{e}{f}})$ :: AB $(=1)$: AI, & $AI=\sqrt{\frac{e+fz^n}{f}}$. Et inſuper $FI=\sqrt{AIq-AFq}=\sqrt{\frac{e+fz^n}{f}-\frac{e}{f}}=z^{\frac{1}{n}}$. Cum autem area curvæ, cujus ordinata eſt $\frac{ez^{\frac{1}{n}-1}}{e+fz^n}$, ſit $=\frac{4}{n}ABE$, eadem area æqualis erit $\frac{2}{n}\times FG\times AB$; & proinde area curvæ, cujus ordinata eſt $\frac{dz^{\frac{1}{n}-1}}{e+fz^n}$, quæ nimirum habetur in *Coteſii* tabula, eſt $=\frac{2d}{ne}\times FG\times AB=\frac{2d}{ne}\times FG\times 1$; exiſtente arcus FG radio, quocum deſcribitur, $=\sqrt{\frac{e}{f}}$, ejus tangente $=z^{\frac{1}{n}}$, ejuſdemque ſecante $=\sqrt{\frac{e+fz^n}{f}}$: & hæc eſt quadratura a *Coteſio* exhibita.



PORRO ducta linea CK, quæ tangens ſit complementi ad quadrantem arcus FG, erit AK complementi ejuſdem ſecans; & cum IF $(z^{\frac{1}{n}})$: AF $(=\sqrt{\frac{e}{f}})$ :: AC $(=\sqrt{\frac{e}{f}})$: CK, erit $CK=\frac{e}{fz^{\frac{1}{n}}}$; & proinde

proinde $AK = \sqrt{ACq + CKq}$ eſt $= \frac{\sqrt{e \times e + fz^{n}}}{fz^{\frac{1}{2}n}}$, quæ ſunt tangens & ſecans arcus exhibitæ *pag.* 64. *libri Coteſii.*

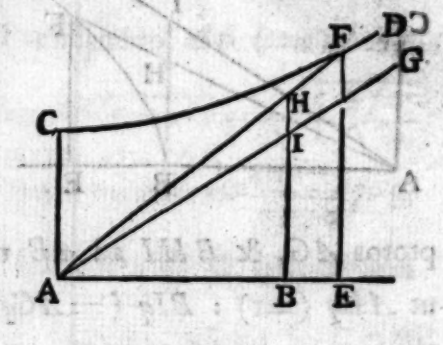
Porro, quando curvæ quadratura ab hyperbola pendet, quod accidit, cum alteruter terminorum in denominatore $e + fz^{n}$ negativus ſit, ſumendi ſunt hyperbolæ ſemiaxes AB, AC, quorum AB unitatem denotet, & alter AC ſit $= \sqrt{\frac{e}{f}}$, deinde his ſemiaxibus deſcribenda eſt hyperbola $B\mathcal{D}$, ſi habeatur in ordinata curvæ metiendæ $e - fz^{n}$; deſcribenda autem eſt iiſdem ſemiaxibus hyperbola $C\mathcal{D}$*, ſi habeatur $-e + fz^{n}$: deinde ſumenda eſt in caſu priori $AE = \sqrt{\frac{e}{e - fz^{n}}}$, & in caſu altero $AE = \sqrt{\frac{e}{-e + fz^{n}}}$, poſtea ordinatim



applicanda eſt EF ducendaque AF; tum area curvæ, cujus ordinata eſt $\frac{ez^{\frac{1}{2}n-1}}{e - fz^{n}}$, æqualis erit $\frac{4}{n} \times ABF$, & area curvæ ordinatam habentis $\frac{ez^{\frac{1}{2}n-1}}{-e + fz^{n}}$ æqualis erit $\frac{4}{n} \times ACF$. Jam ut *Coteſii* quadratura hinc derivetur, ducatur aſymptotos AG, & BHI ad AB normalis. Deinde erit in caſu priori ut ABq $(=1)$: BIq $(= ACq = \frac{e}{f})$ :: AEq $(= \frac{e}{e - fz^{n}}) - ABq$ $(=1)$: EFq; unde $EF =$ eſt $\sqrt{\frac{ez^{n}}{e - fz^{n}}}$; ſed in caſu altero erit ut ABq : BIq :: AEq $(= \frac{e}{-e + fz^{n}}) + ABq$: EFq, adeo ut hic $EF =$ erit $\sqrt{\frac{ez^{n}}{-e + fz^{n}}}$. Porro ut AE : AB $(=1)$ :: EF : BH; unde BH in caſu utroque erit $= \frac{EF}{AE} = z^{\frac{1}{2}n}$. Jam pro more loquendi a *Coteſio* adhibito ſpatium ABF in caſu priori menſura eſt ſubduplicatæ rationis inter $BI + BH$ & $BI - BH$ †; vel men-

* Vid. Fig. pag. ſequent. † Logometr. Prop. 4.

ſura rationis quam $BI+BH$ habet ad $\sqrt{BIq-BHq}$, id eſt rationis quam $\sqrt{\frac{e}{f}+z^{\frac{1}{\eta}}}$ habet ad $\sqrt{\frac{e-fz^{\eta}}{f}}$, triangulo $ABI=\frac{1}{2}AB\times BI$, hoc eſt $\frac{1}{2}\sqrt{\frac{e}{f}}$, exiſtente modulo hujus menſuræ; item in caſu altero ſpatium ACF eſt menſura rationis, quam $BI+BH$ habet ad $\sqrt{BHq-BIq}$, vel rationis, quam $\sqrt{\frac{e}{f}+z^{\frac{1}{\eta}}}$ habet ad $\sqrt{\frac{-e+fz^{\eta}}{f}}$, triangulo $ABI=\frac{1}{2}\sqrt{\frac{e}{f}}$ exiſtente iſtius menſuræ modulo. Quoniam igitur area curvæ, cujus ordinata eſt $\frac{ez^{\frac{1}{\eta}-1}}{e-fz^{\eta}}$, eſt $=\frac{4}{\eta}ABF$, & area curvæ ordinatam habentis $\frac{ez^{\frac{1}{\eta}-1}}{-e+fz^{\eta}}$ æqualis $\frac{4}{\eta}ACF$, area prior æqualis eſt quantitati quam *Coteſius* ſic exhibuiſſet $\frac{2}{\eta}\sqrt{\frac{e}{f}}\left|\frac{\sqrt{\frac{e}{f}+z^{\frac{1}{\eta}}}}{\sqrt{\frac{e-fz^{\eta}}{f}}}\right.$; &



area poſterior $=\frac{2}{\eta}\sqrt{\frac{e}{f}}\left|\frac{\sqrt{\frac{e}{f}+z^{\frac{1}{\eta}}}}{\sqrt{\frac{-e+fz^{\eta}}{f}}}\right.$;

ideoque area curvæ, cujus ordinatata eſt $\frac{dz^{\frac{1}{\eta}-1}}{e-fz^{\eta}}$, erit $=\frac{2d}{\eta e}\sqrt{\frac{e}{f}}\left|\frac{\sqrt{\frac{e}{f}+z^{\frac{1}{\eta}}}}{\sqrt{\frac{e-fz^{\eta}}{f}}}\right.$, & area curvæ, cujus ordinata eſt $\frac{dz^{\frac{1}{\eta}-1}}{-e+fz^{\eta}}$, æqualis erit $\frac{2d}{\eta e}\sqrt{\frac{e}{f}}\left|\frac{\sqrt{\frac{e}{f}+z^{\frac{1}{\eta}}}}{\sqrt{\frac{-e+fz^{\eta}}{f}}}\right.$. In his ſi pro $\sqrt{\frac{e}{f}}$, vel ſumptis quantitatibus

tatibus e & f cum ſignis ſuis, pro $\sqrt{-\frac{e}{f}}$, ponamus R; pro $z^{\frac{1}{n}}$ ponamus T; & pro differentia inter TT & RR ponamus SS, utraque arearum præcedentium ſic exprimetur $\frac{2d}{ne}$ R $\left|\frac{R+T}{S}\right.$. Obſervandum autem eſt in caſu priori, ubi T minor eſt quam R, quantitatem SS ex *Coteſii* tabula deſumptam eſſe negativam, & ipſam S quantitatem impoſſibilem; ſed quandocunque id contingit, quantitatem SS adhiberi poſſe ſigno mutato peritiſſimus *Coteſii* operis editor admonuit *.

Præterea ut $z^{\frac{1}{n}} : \sqrt{\frac{e}{f}} :: \sqrt{\frac{e}{f}} : \frac{e}{fz^{\frac{1}{n}}}$. Unde in caſu priori erit $BI + BH : \sqrt{BIq - BHq} :: \sqrt{\frac{e}{f}} + \frac{e}{fz^{\frac{1}{n}}} : \frac{\sqrt{e \times e - fz^2}}{fz^{\frac{1}{n}}}$; & in caſu altero $BI + BH : \sqrt{BHq - BIq} :: \sqrt{\frac{e}{f}} + \frac{e}{fz^{\frac{1}{n}}} : \frac{\sqrt{e \times -e + fz^2}}{fz^{\frac{1}{n}}}$. Hinc area curvæ ordinatam habentis $\frac{cz^{\frac{1}{n}-1}}{e - fz^2}$ æqualis erit $\frac{2}{n} \times \sqrt{\frac{e}{f}} \left| \frac{\sqrt{\frac{e}{f}} + \frac{e}{fz^{\frac{1}{n}}}}{\frac{\sqrt{e \times e - fz^2}}{fz^{\frac{1}{n}}}} \right.$, & area curvæ ordinatam habentis $\frac{ez^{\frac{1}{n}-1}}{-e + fz^2}$ æqualis $\frac{2}{n} \sqrt{\frac{e}{f}} \left| \frac{\sqrt{\frac{e}{f}} + \frac{e}{fz^{\frac{1}{n}}}}{\frac{\sqrt{e \times -e + fz^2}}{fz^{\frac{1}{n}}}} \right.$: denique area curvæ, cujus ordinata eſt $\frac{dz^{\frac{1}{n}-1}}{e - fz^2}$, erit $= \frac{2d}{ne} \sqrt{\frac{e}{f}} \left| \frac{\sqrt{\frac{e}{f}} + \frac{e}{fz^{\frac{1}{n}}}}{\frac{\sqrt{e \times e - fz^2}}{fz^{\frac{1}{n}}}} \right.$, & area curvæ ordinatam habentis $\frac{dz^{\frac{1}{n}-1}}{-e + fz^2}$ erit $= \frac{2d}{ne} \sqrt{\frac{e}{f}} \left| \frac{\sqrt{\frac{e}{f}} + \frac{e}{fz^{\frac{1}{n}}}}{\frac{\sqrt{e \times -e + fz^2}}{fz^{\frac{1}{n}}}} \right.$. Et hic exhibentur quadraturæ a *pagina* 64 *libri Coteſii* petendæ.

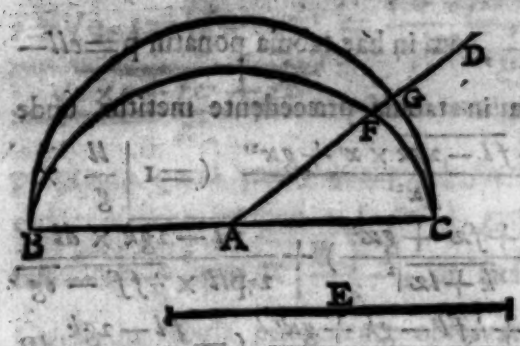
* Not. in Harmon. Menſur. pag. 99.

AREA autem curvæ sic inventa, cujus ordinata est $\frac{dz^{\frac{1}{n}-1}}{e+fz^n}$, tabulæ pars superior deducitur ab hac area ope *prop. 7. quadrat. Newtoni*; & cum $\frac{dz^{\frac{1}{n}-1}}{e+fz^n}$ fit $= \frac{dz^{\frac{1}{n}+n-1}}{f+ez^n}$, ab area curvæ, cujus ordinata est $\frac{dz^{\frac{1}{n}+n-1}}{f+ez^n}$, tabulæ pars inferior ejusdem propositionis ope derivatur.

TABULÆ tertia, quarta, quinta & sexta eodem modo facile derivantur a formis *Newtoni*. Partes superiores septimæ & nonæ, & inferiores octavæ & decimæ non difficilius deducuntur a *Newtoni* placitis. E quibus una cum tabulis tertia & quarta, quinta & sexta, reliquæ quoque harum tabularum partes deducuntur ope *prop.* 8. *quadr. Newtoni*. Pro exemplo invenienda sit area curvæ, cujus ordinata est $\frac{dz^{-1}\sqrt{e+fz^2}}{g+hz^2}$. Ponatur hæc area æqualis duarum curvarum areis, quarum ordinatæ sunt $\frac{pz^{-1}\sqrt{e+fz^2}}{g+hz^2}$, & $qz^{-1}\sqrt{e+fz^2}$; unde areæ curvæ æqualis erit, cujus ordinata est $\frac{p+qh\times z^{-1}+qgz^{-1}\times\sqrt{e+fz^2}}{g+hz^2}$; ideoque ponendum est $p+qh=0$, & $qg=d$; id est $q=\frac{d}{g}$, & $p=-\frac{dh}{g}$. Est igitur area quæsita æqualis excessui areæ curvæ, cujus ordinata est $\frac{d}{g}z^{-1}\sqrt{e+fz^2}$, præ area curvæ, cujus ordinata est $\frac{\frac{dh}{g}z^{-1}\sqrt{e+fz^2}}{g+hz^2}$.

PARTES harum tabularum a *Newtono* omissæ etiam alio modo suppleri possunt; verbi causa, si ponatur $\sqrt{g+hz^2}=x$, curva, cujus ordinata est $\frac{dz^{\frac{1}{n}-1}\sqrt{e+fz^n}}{g+hz^n}$, quæ apparebit eadem ac præcedens, mutando solummodo indicem n ex affirmativo in negativum, ream habet æqualem areæ curvæ, cujus abscissa est x & ordinata $\frac{2vd}{nh}x^{-1}\sqrt{\frac{eh-fg+fx^{2v}}{-g+x^{2v}}}$, quæ est curva in ultima formarum *Newtoni*; & hoc reducendi modo theoremata inveni ad nodos lunæ pertinentia, quorum demonstrationes olim tibi impertivi. Horum theorematum, ut forsan recorderis, hoc primum fuit. Designet A centrum terræ,

BC



BC lineam nodorum, *AD* lineam rectam ad ſolem inclinatam, & ſumatur quoque linea *E* ad *BC* in ratione velocitatis ſolis ad aggregatum hujus velocitatis & velocitatis mediæ nodorum, quando ſunt in quadraturis ad ſolem; & axe *BC* latereque recto *E* ellipſis deſcribatur *CFB*; tum tempus, quo ſol & linea nodorum ſe ſeparaverunt ſpatium anguli ſub *CAD*, ſemper erit ut ſpatium ellipſi comprehenſum *CAF*. Theorema ſecundum hoc erat: Semicirculo *CGB* deſcripto, motus verus nodi, cum angulus ſub *CAD* deſcribitur ſeparatione ſolis ab eo, erit ut ſpatium *CFG*. Theoremate alio oſtenſum fuit angulum ſub *DAF* in *Prop.* 33. *libri* 3. *Philoſ. Nat. Princip. Math.* eſſe æquationem nodorum veram.

TABULA undecima eſt eadem ac forma ultima *Newtoni*. Tabula duodecima invenitur ponendo $k+lz^{\nu}=x^{\nu}$; hac enim ratione curva, cujus abſciſſa eſt z & ordinata $\frac{dz^{\nu-1}}{k+lz^{\nu}}\sqrt{\frac{e+fz^{\nu}}{g+bz^{\nu}}}$, *prop.* 9. *quadrat. Newtoni* invenietur habere aream æqualem areæ curvæ, cujus abſciſſa eſt x & ordinata $\frac{\nu d}{\eta l}x^{-1}\sqrt{\frac{el-kf+fx^{\nu}}{lg-bk+bx^{\nu}}}$. Et ordinata $\frac{dz^{-1}}{k+lz^{\nu}}\sqrt{\frac{e+fz^{\nu}}{g+bz^{\nu}}}$ eſt $=\frac{dz^{-\nu-1}}{l+kz^{-\nu}}\sqrt{\frac{f+ez^{-\nu}}{b+gz^{-\nu}}}$, quæ eadem eſt cum priori, quando indicis ν ſignum eſt negativum.

TABULÆ 13, 15, 16, eædem ſunt ac formæ 5, 7, & 8 *Newtoni*, & curvarum in iis exhibitarum quadratura haud difficilius deducitur a *Newtoni* præceptis, quam tabula ſecunda inde deducta fuit.

PORRO quadratura curvarum in tabulis decima quarta & decima octava invenitur ponendo $k+lz^{\nu}=x^{\nu}$.

HAC ratione in tabula 14 curva, cujus abſciſſa eſt z & ordinata $\frac{dz^{\nu-1}}{k+lz^{\nu}\times e+fz^{\nu}+gz^{2\nu}}$, æqualis invenietur areæ curvæ, cujus abſciſſa eſt x & ordinata $\frac{\frac{\nu}{\eta}dlx^{-1}}{ell-fkl+gkk+fl-2gk\times x^{\nu}+gx^{2\nu}}$,

vel $= \frac{\frac{1}{\eta} dlx^{-1}}{p + fl - 2gk \times x^{\eta} + gx^{2\eta}}$, cum in hac tabula ponatur $p = ell - fkl + gkk$. Hæc autem curva in tabula præcedente metitur, unde invenies eam $= \frac{-dl}{2\eta p} \times 1 \left| \frac{p + fl - 2gk \times x^{\eta} + gx^{2\eta}}{gx^{2\eta}} \right.$ $(= 1 \left| \frac{ll}{g} \right. \times \frac{e + fz^{\eta} + gz^{2\eta}}{k + lz^{\eta}} \Big|^2 = 1 \left| \frac{ll}{g} + 1 \right| \frac{e + fz^{\eta} + gz^{2\eta}}{k + lz^{\eta}} \Big|^2) + \frac{fl - 2gk \times dt}{2\eta pll \times \frac{1}{4} ff - eg}$ $\times l \sqrt{\frac{1}{4} ff - eg} \left| \frac{l \sqrt{\frac{1}{4} ff - eg} + \frac{1}{2} fl - gk + gx^{\eta}}{\sqrt{g} \times p + fl - 2gk \times x^{\eta} + gx^{2\eta}} \right.$ $(= \frac{fl - 2gk}{2\eta pq} dR \left| \frac{R + T}{S} \right.)$, positis $q = \frac{1}{4} ff - eg$, $R = \sqrt{q}$, $T = \frac{1}{2} f + gz^{\eta}$, & $S = \sqrt{g \times e + fz^{\eta} + gz^{2\eta}}$: vel neglecta data quantitate $1 \left| \frac{ll}{g} \right.$, erit hæc area $= \frac{-dl}{2\eta p} \times 1 \left| \frac{e + fz^{\eta} + gx^{2\eta}}{k + lz^{\eta}} \Big|^2 + \frac{fl - 2gk}{2\eta pq} dR \left| \frac{R + T}{S} \right.$.

DEINDE area curvæ, cujus abscissa est x & ordinata $\frac{dz^{2\eta - 1}}{k + lz^{\eta} \times e + fz^{\eta} + gz^{2\eta}}$, æqualis erit areæ curvæ, cujus abscissa est x & ordinata $\frac{dx^{\eta - 1} - dkx^{-1}}{p + fl - 2gk \times x^{\eta} + gx^{2\eta}}$; adeo ut curvæ præcedentis quadratura, una cum curvæ quadratura ordinatam habentis $\frac{dx^{\eta - 1}}{p + fl - 2gk \times x^{\eta} + gx^{2\eta}}$, habetur curvæ quadratura, cujus ordinata est $\frac{dz^{2\eta - 1}}{k + lz^{\eta} \times e + fz^{\eta} + gz^{2\eta}}$. Vel mutando indicis η signum eandem quadraturam paulo aliter invenire licet; est enim ordinata $\frac{dz^{2\eta - 1}}{k + lz^{\eta} \times e + fz^{\eta} + gz^{2\eta}} = \frac{dz^{-\eta - 1}}{l + kz^{-\eta} \times g + fz^{-\eta} + ez^{-2\eta}}$; unde conferendo hanc ordinatam cum ordinata priori illius quadratura hujus quoque quadraturam habes, nimirum invenies hujus curvæ aream, æqualem $\frac{dk}{2\eta p} \times 1 \left| \frac{g + fz^{-\eta} + ez^{-2\eta}}{l + kz^{-\eta}} \Big|^2 + \frac{2el - fk}{2\eta pq} dr \left| \frac{r + t}{s} \right.$, positis

$r = \sqrt{q}$

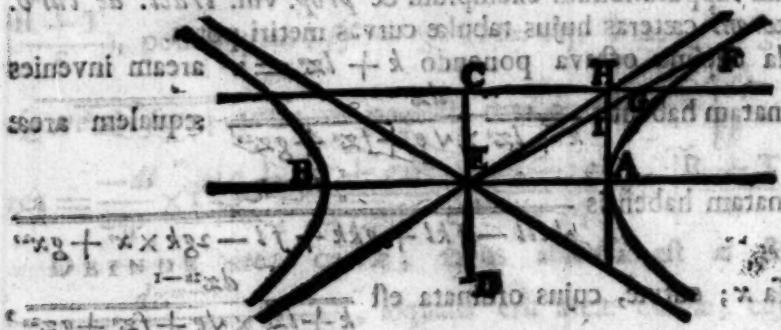
$r = \sqrt{q}$, $t = \frac{1}{2}f + ez^{-1} = \frac{e + \frac{1}{2}fz}{z^2}$, $s = \sqrt{e \times g + fz^{-1} + ez^{-2}}$ $= \frac{\sqrt{e \times e + fz + gz^2}}{z^2}$. Hæc autem area æqualis est præcedenti, propterea quod ratio, quam $g + fz^{-1} + ez^{-2}$ habet ad $l + kz^{-n}$, eadem est cum ratione $e + fz + gz^2$ ad $k + lz^n$, & usurpari potest $-n \left| \frac{r+t}{s} \right.$ pro $R \left| \frac{R+T}{S} \right.$, ut facile supputando invenies; & a *Cotesio* in tabula præcedente observatur, a qua tabula hæc decima quarta tota dependet.

Ad harum supputationum exemplum & *prop.* viii. *tract. de curv. quadr. Newtoni* cæteras hujus tabulæ curvas metiri potes.

In tabula decima octava ponendo $k + lz^n = x^n$ aream invenies curvæ ordinatam habentis $\frac{dx}{k + lz^n \times \sqrt{e + fz + gz^2}}$ æqualem aream curvæ ordinatam habentis $\frac{dx^{2n-1}}{\sqrt{ell - fkl + gkk + fl - 2gk \times x^n + gx^{2n}}}$ cum abscissa x; curvæ, cujus ordinata est $\frac{dx^{2n-1}}{k + lz^n \times \sqrt{e + fz + gz^2}}$, aream aream curvæ ordinatam $\frac{\frac{rd}{nl} x^{n-1} - \frac{rk}{nl} dx^{-1}}{\sqrt{ell - fkl + gkk + fl - 2gk \times x^n + gx^{2n}}}$ habentis æqualem; item ordinata $\frac{dz^{-n-1}}{k + lz^{-n} \times \sqrt{e + fz + gz^2}}$ est $\frac{dz^{-n-1}}{l + kz^{-n} \times \sqrt{g + fz^{-1} + ez^{-2}}}$; unde curvæ hac ordinata definitæ mensura deducitur a mensura curvæ, cujus ordinata est $\frac{dz^{n-2}}{k + lz^n \times \sqrt{e + fz + gz^2}}$, quæ ordinata eandem habet formam.

Hac ratione aream invenies $= \frac{1}{nkp} dR \left| \frac{R+T}{S} \right. - \frac{1}{nek} d\varrho \left| \frac{\varrho + \tau}{\sigma} \right.$; ubi ϱ, τ, σ, R, S eadem denotant, ac in auctoris nostri tabula, sed nostra quantitas quam T denotat illius — T æqualis erit. Unde si quantitati

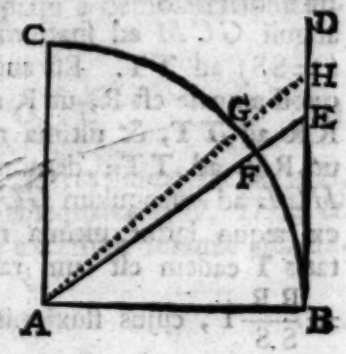
cati T cum *Coteſio* contrarium tribuamus ſignum, erit area $= \frac{l}{\eta k p} dR \left| \frac{R+T}{S} - \frac{1}{\eta e k} d\varrho \right| \frac{\varrho+\tau}{\sigma}$; mutato enim ſigno quantitatis T areæ quoque, cui pertinet, ſignum, magnitudine ſua ſervata, mutatur. Hoc autem in circulo manifeſtum eſt, ſcilicet arcus ſignum mutari, tangentis ſuæ mutato ſigno. In hyperbola idem demonſtrabimus modo ſequenti. Sint hyperbolæ oppoſitæ *A*, *B*, quarum axis tranſverſus ſit *AB*, rectus *CD*, centrum *E*. Sit $AE \times EC = R$, ſumpto autem ad libitum puncto *F*, ducatur *EF*, item linea *CHG* axi tranſverſo *AB* parallela. Jam cum *Coteſius* ſpatium *AEF* menſuram appellat rationis, quam $CH + CG$ habet ad $\sqrt{CGq - CHq}$, dimidio rectangulo



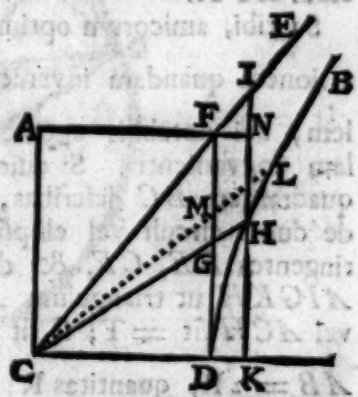
ſub $CEA = \frac{1}{2}R$ exiſtente iſtius menſuræ modulo; duplum ſpatium illud his notis $R \left| \frac{R+T}{S} \right.$ denotatur, ita ut ratio, R + T ad S eadem ſit cum ratione $CH + CG$ ad $\sqrt{CGq - CHq}$. Quando igitur T major eſt quam R erit *CG* ad *CH* ut T ad R, quando autem T minor eſt quam R erit *CH* ad *CG* ut T ad R. Jam in caſu utroque manifeſtum eſt, quando quantitatis T ſignum mutatur, ab altera parte puncti *C* lineam *CG* ſumendam eſſe, ſpatiumque hyperbolicum ab altera parte lineæ *EC* incidere; ideoque ſignum habere mutatum; ſed ejuſdem eſſe magnitudinis, ſi linea *CG* ejuſdem longitudinis ſit. Idem quoque demonſtrari poteſt ducendo *AIH* & oſtendendo *AI* & *AH* eam inter ſe rationem habere, quam R & T; unde etiam intelligitur ſpatium hyperbolicum ab altera parte lineæ *EA* cadere, id eſt diverſum habere ſignum, quando quantitatis T ſignum mutatur.

Denique, ut ea ad finem perducam, quæ de his prioribus in *Coteſii* opere tabulis ad te ſcribere inſtitui, nihil amplius reſtat, quam ut te moneam tabulam decimam ſeptimam a tabula proxime ſequente deduci ope *Propoſ. octav. tract. de curv. quadrat. Newtoni.* Niſi operæ pretium quoque ſit demonſtrare fluxionem quantitatis $R\left|\frac{R+T}{S}\right.$ in circulo eſſe $=\frac{RR}{SS}\dot{T}$, & in hyperbola $=-\frac{RR}{SS}\dot{T}$; * quo cognito quadraturarum hic a *Coteſio* exhibitarum examinare potes veritatem †.

Ut autem hoc in circulo faciamus, radio $AB=R$ deſcribatur circuli quadrans ABC & contingens ducatur $B\mathcal{D}$, in qua ſumatur $BE=T$, ducta AFE. Deinde ducatur AGH, & diminuta HE ſine limite erit ratio AEq ad AFq vel ratio SS ad RR eadem cum ultima ratione trianguli HAE, ad ſpatium GAF, vel cum ultima ratione $\frac{1}{2}AB\times HE$ ad $\frac{1}{2}AF\times FG$, id eſt, cum ultima ratione HE ad FG: eſt autem ultima ratio HE ad FG eadem ac ratio fluxionis BE vel † ad fluxionem arcus BF; fluxio igitur arcus BF quantitate $R\left|\frac{R+T}{S}\right.$ deſignati æqualis eſt $\frac{RR}{SS}\dot{T}$.



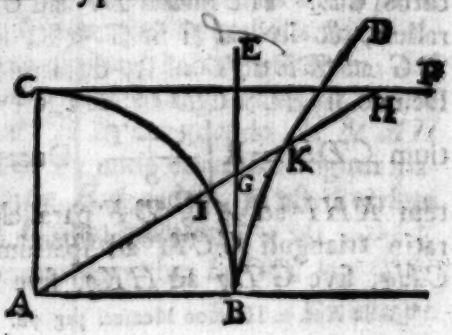
Ita res in circulo oſtenditur, nec diverſa eſt demonſtratio, quam eruditiſſimus *Coteſii* operis editor in notis ſuis dedit ‡. Ut autem idem in hyperbola efficiatur, ſit hyperbola $\mathcal{D}B$, cui ſemiaxis tranſverſus ſit $C\mathcal{D}$, ſemiaxis rectus CA, aſymptotos CE. Hic ducta $\mathcal{D}F$ ad CA parallela, & in ea, ſi ſit $T<R$, ſumpta $\mathcal{D}G$ ad $\mathcal{D}F$ ut T ad R, ducatur CGH. Deinde ſi $AC\times C\mathcal{D}$ ſit $=2R$ erit ſpatium $C\mathcal{D}H=R\left|\frac{R+T}{S}\right.$. Ducatur autem KHI ad AC, $\mathcal{D}F$ parallela, & CML. Deinde erit ultima ratio trianguli GCM ad ſpatium HCL eadem ac ratio CGq ad CHq, ſive $G\mathcal{D}q$ ad HKq, ſive $\mathcal{D}Fq$ ad KIq, ſive $\mathcal{D}Fq-\mathcal{D}Gq$



* Vide Not. in Harmon Menſur. pag. 98. † Vid. ibid. pag. 100. ‡ Pag. 98.

ad $IKq - HKq$ $(= \mathcal{D}Fq)$ ſive denique eadem ac ratio RR — TT (= — SS) ad RR. Jam cum ſit $G\mathcal{D} : \mathcal{D}F$ (:: $G\mathcal{D} \times \mathcal{D}C : \mathcal{D}F \times \mathcal{D}C$) :: T : R, & $\mathcal{D}C \times AC = \mathcal{D}C \times \mathcal{D}F$ ſit = 2 R, erit triangulum $C\mathcal{D}G$ = T, & ultima ratio GCM ad HCL, quæ eadem eſt cum ratione — SS ad RR, eadem erit cum ratione fluxionis trianguli $C\mathcal{D}G$ ſive $\dot{T}$, ad fluxionem ſpatii $C\mathcal{D}H$; ideoque illa fluxio $= -\frac{RR}{SS}\dot{T}$, nimirum ſi ſit T < R. Idem autem jam demonſtrabimus, quando T > eſt quam R. In hoc caſu, quando ſpatium $C\mathcal{D}H$ eſt $= R\left|\frac{R+T}{S}\right.$, erit $\mathcal{D}G$ ad $\mathcal{D}F$ ut R ad T, & ultima ratio trianguli GCM ad ſpatium HCL eadem cum ratione TT — RR (= SS) ad TT. Eſt autem triangulum $G\mathcal{D}C$ ad triangulum $F\mathcal{D}C$, quod æquale eſt R, ut R ad T, triangulum igitur $G\mathcal{D}C$ erit ad T ut RR ad TT, & ultima ratio GCM ad incrementum quantitatis T ut RR ad TT; demonſtravimus autem ultimam rationem ſpatii HCL ad triangulum GCM eandem eſſe cum ratione TT ad SS; ex æquo igitur ultima ratio ſpatii HCL ad incrementum quantitatis T eadem eſt cum ratione RR ad SS, & fluxio ſpatii $C\mathcal{D}H$ $= \frac{RR}{SS}\dot{T}$, cujus fluxionis ſignum mutare oportet, propterea quod area hic decreſcit, quando T increſcit, & contra. Hæc autem omnia, & eodem plane modo, demonſtrari poſſunt ducendo lineam rectam AFN.

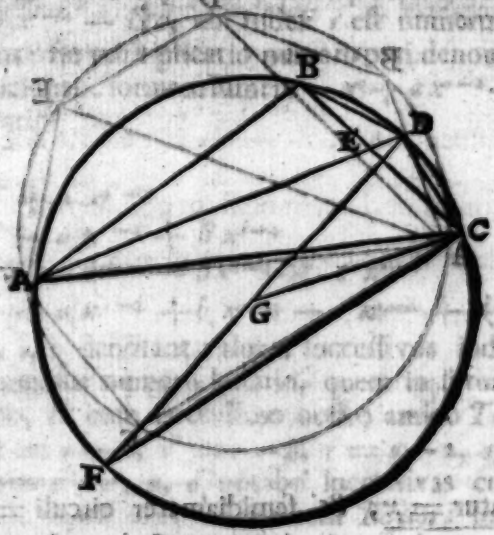
Si tibi, amicorum optime, moleſtum non fuerit his diutius detineri, rationem quandam inveniendi quantitatem $R\left|\frac{R+T}{S}\right.$ apponere vellem, ubi notabilis apparet inter circulum vel ellipſin & hyperbolam convenientia. Si enim ſemiaxibus AB, AC circuli vel ellipſis quadrantem BC deſcribas, & etiam hyperbolam $B\mathcal{D}$; & deinde ducas circuli vel ellipſis contingentes BE, CF, & denique $AIGKH$ ut triangulum ABG, vel ACH ſit = T; ſi ſit $AC \times AB$ = 2 R, quantitas $R\left|\frac{R+T}{S}\right.$ erit in circulo = ſectori ABI vel ACI, & in hyperbola = ſectori ABK.



JAM progrediar ad alteram ex partibus, in quas distribui magni *Cotesii* inventa, in qua hæc doctrina ulterius quam in priori excolitur. Inventa hac parte comprehensa ad duos locos generales reduci possunt; quorum alter curvas ordinatas rationales habentes continet, alter autem curvas cum ordinatis irrationalibus sive surdis. Curvæ ordinatarum rationalium primum considerari merentur; quarum metiendi ratio oritur ex methodo *Cotesii* ad quadraturam omnium curvarum, quarum ordinatæ sunt fractiones rationales cum denominatoribus binomiis. Et hæc methodus eo diligentius excutienda est propter eximiam elegantiam in methodis generalibus raro inveniendam. Sed cum hæc pars ejus inventi posita sit in theoremate, quod invenit ad assignandos divisores omnium ejusmodi binomiorum, incipiam a demonstratione hujus theorematis propositionibus sequentibus.

PROPOSITIO I.

SI in circulo *ABC* ab aliquo puncto *A* tres lineæ rectæ *AB*, *AD*, *AC* ducantur intercipientes arcus æquales *BD*, *DC*; & chordæ quoque *BD*, *DC* ducantur; dico excessum summæ quadra-



torum ab *AB*, *AC* præ duplo quadrato a *BD* vel *DC* esse ad quadratum ab *AD*, ut excessus dupli quadrati a radio præ quadrato a *BD* vel *DC* ad radii quadratum.

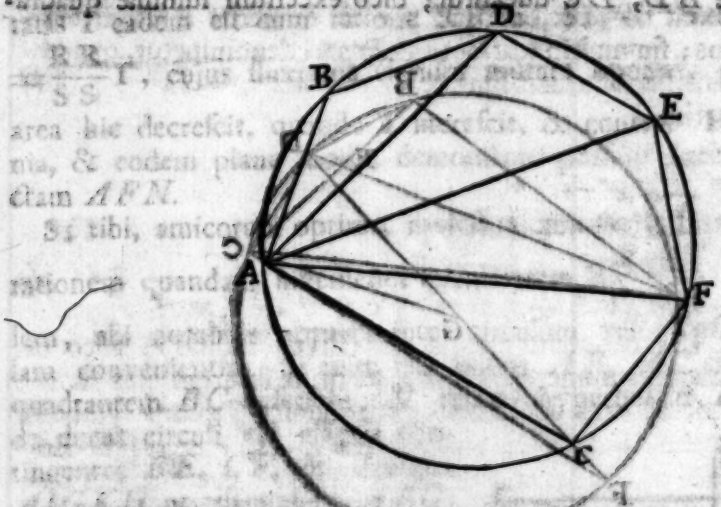
DUCANTUR

DUCANTUR BEC, diameter DF, ex centro G linea GC, & jungatur CF. Triangula DAB, DBE ſunt ſimilia, & etiam triangula DAC, BAE. Ideoque $AD \times DE$ eſt $= BDq$, & $DA \times AE$ eſt $= AB \times AC$, ut etiam $ADq = BDq + AB \times AC$. Porro $AB + AC : AD :: BC : BD^{*} :: CF : FG$, propterea quod linea AD dividit angulum ſub BAC in duas partes æquales, & quia triangula iſoſcelia DBC, GFC, ſunt ſimilia: igitur $\overline{AB+AC}q : ADq$ $(= BDq + AB \times AC) :: CFq\ (= 4FGq - DCq) : FGq$, & bis dividendo $\overline{AB+AC}q - 2AB \times AC - 2BDq\ (= ABq + ACq - 2BDq) : ADq :: 2FGq - DCq\ (BDq) : FGq$, Q. E. D.

PROPOSITIO II.

CHORDA arcus alicujus data invenire chordam cujuſvis multiplicis iſtius arcus.

IN circulo ABC ſint arcus AB, BD, DE, EF, FC æquales, chorda AB data, invenire AD, AE, AF, AC.



SI AB ponatur $= x$, & ſemidiameter circuli $= 1$, notum eſt $ADq =$ eſſe $\overline{-xx+2} \times xx + 2xx$. Sed *prop. præcedenti* $ABq + AEq - 2BDq : ADq :: -xx + 2 : 1$. Adeo ut AEq erit $= ADq \times \overline{-xx+2} + 2BDq - ABq$. Et eadem ratione AFq erit $= AEq \times \overline{-xx+2} + 2BDq\ (2xx) - ADq$ &c.

* Vid. Euclid. Dat. prop. 94.

Quamobrem

Quamobrem

ABq eſt $= xx$

$ADq = -x^4 + 4x^3$

$AEq = x^6 - 6x^4 + 9x^2$

$AFq = -x^8 + 8x^6 - 20x^4 + 16x^2$

$ACq = x^{10} - 10x^8 + 35x^6 - 50x^4 + 25x^2$

Ratio commodiſſima generatim inveniendi coefficientes terminorum in his chordarum valoribus a *Methodo Incrementorum* petenda eſt eximii Geometræ *Taylori*, quem virum humaniſſimum communemque amicum utrique noſtrum unice diligimus. Ille omnium primus in ſeriebus inveniendis illuſtravit methodi *Newtonianæ*, quæ vocatur differentialis, utilitatem *, cujus admirabilis inventi uſus omnes haud facile quis enumeret. Sed & aliam quoque rationem aliquanto diverſam ſierierum coefficientes inveniendi vir ille acutiſſimus docuit; quam hic ſequi oportet.

Primum autem obſervandum eſt coefficientes ultimorum terminorum in omnibus his valoribus quadrata eſſe numerorum ab unitate ordine naturali ſe invicem ſequentium; deinde valorem quadrati chordæ arcus cujuſvis multiplicis, ſi multiplicatio ſit impar, hanc formam induere $x^{\nu} - ax^{\nu-2} + bx^{\nu-4} - \&c$, ubi index ν eſt numerus multiplicationem bis exprimens; ſin multiplicatio numero pari denominetur, quadratum arcus multiplicis hanc formam ſumere $- x^{\nu} + ax^{\nu-2} - bx^{\nu-4} + \&c$; poſſe igitur generatim

$ABq = x^{\nu}$

$ADq = -x^{\nu} + a^{\nu} x^{\nu-2}$

$AEq = x^{\nu} - a^{\prime} x^{\nu-2} + b^{\prime} x^{\nu-4}$

$AFq = -x^{\nu} + a^{\prime\prime} x^{\nu-2} - b^{\prime\prime} x^{\nu-4} + c^{\prime\prime} x^{\nu-6}$

$ACq = x^{\nu} - a_{\prime\prime\prime} x^{\nu-2} + b_{\prime\prime\prime} x^{\nu-4} - c_{\prime\prime\prime} x^{\nu-6} + d_{\prime\prime\prime} x^{\nu-8}$

Hic ν''', ν'', ν', ν, $\nu_{\prime}$ denotant valores ſucceſſivos indicis ν; adeo ut ſicut ν perpetuo augetur numero binario, quem in futurum ejus incrementum nominabo, & cum doctiſſimo noſtro amico *Tayloro* ſymbolo ν notabo, ν''' erit $= \nu - 6$, $\nu'' = \nu - 4$, $\nu' = \nu - 2$, $\nu_{\prime} = \nu + 2$. Eodem modo ſymbolis a'', a', a, $a_{\prime}$ notabo ſucceſſivas coefficientes termini ſecundi, & ſic de cæteris, ad mentem *Taylori* in Methodo ſua Incrementorum.

Sed jam ex modo, quo hæc quadrata chordarum inveniuntur, eſt $a_{\prime} = a + 2$, ideoque $a_{\prime} - a = a = 2 = \nu$, unde ex forma in *pag.* 19. *Method. Increment.* ad inveniendum integrale ex incremento dato,

* Vid. Method. Incremen. pag. 55.

erit $a = A + \nu$, ubi A est quantitas assumpta & determinanda ex valore a in casu aliquo particulari. At ubi a est terminus ultimus in valore quadrati chordæ, est $= \nu$; igitur A est $= 0$, & a semper est $= \nu$. Rursus b, est $= b + 2a - 1$, & $b, - b = b' = 2a - 1 = 2\nu - 1 = \nu - \frac{1}{2}\nu$. Eadem igitur forma *Taylori* b erit $= A + \frac{1}{2}w' - \frac{1}{2}x$, existente A quantitate assumpta ut antea. Sed $\nu' = \nu - 2$, unde b est $= A + \nu \times \frac{\nu - 3}{2}$, & quando b est terminus ultimus in valore quadrati chordæ, est $= \nu \times \frac{\nu - 3}{2}$, ideoque A est $= 0$. Porro c, est $= c + 2b - a'$, & $c, - c = c'$ est $= 2b - a' = w' - \nu - a'$. Sed $a' + 2 = a$, & $a' = a - 2$; igitur cum a sit $= \nu$, c est $= w' - 2\nu + 2 = \frac{1}{2}w'\nu - \nu + 1$; unde forma prædicta c est $= A + \frac{1}{2 \times 3} w'\nu'' - \frac{1}{2}w'\nu + \nu = A + \nu \times \frac{\nu - 2}{2} \times \frac{\nu - 4}{3} - \frac{\nu - 2}{2} + 1 = A + \nu \times \frac{\nu - 4}{2} \times \frac{\nu - 5}{3}$, & in hoc casu A etiam erit $= 0$. Hinc apparet, si x denotet chordam arcus, quadratum chordæ cujusvis arcus hujus multiplicis, & si $\frac{1}{2}\nu$ sit numerus multiplicationem denotans, erit $x^\nu - \nu x^{\nu-2} + \nu \times \frac{\nu - 3}{2} x^{\nu-4} - \nu \times \frac{\nu - 4}{2} \times \frac{\nu - 5}{3} x^{\nu-6}$ &c; si multiplicatio sit impar, sin numero pari denominetur, quadratum istius chordæ erit $= - x^\nu + \nu x^{\nu-2} - \nu \times \frac{\nu - 3}{2} x^{\nu-4} + \nu \times \frac{\nu - 4}{2} \times \frac{\nu - 5}{3} x^{\nu-6}$ &c; si quoties arcus multiplicatur, tot harum serierum termini sumantur.

COROLLARIUM I.

Si radix quadratica hujus seriei $x^\nu - \nu x^{\nu-2} + \nu \times \frac{\nu - 3}{2} x^{\nu-4} - \nu \times \frac{\nu - 4}{2} \times \frac{\nu - 5}{3} x^{\nu-6} +$ &c; ubi terminorum numerus semper est impar, extrahatur, ea radix eandem formam induet, adeo ut si λ sit $= \frac{1}{2}\nu$, ea radix erit $\pm x^\lambda \mp \lambda x^{\lambda-2} \pm \lambda \times \frac{\lambda - 3}{2} x^{\lambda-4} \mp \lambda \times \frac{\lambda - 4}{2} \times \frac{\lambda - 5}{3}$

$\frac{\lambda-5}{3}x^{\lambda-6} \pm$ &c; ita ut quando arcus multiplicatur numero impari, si λ denotet numerum multiplicationis, chorda multiplicis exhibebitur terminis hujus seriei numero æqualibus $\frac{\lambda+1}{2}$ ea lege, ut terminus ultimus semper affirmativus sit.

COROLLARIUM II.

Si hæc series $-x^2+1x^{r-1}-r\times\frac{r-3}{2}x^{r-4}+r\times\frac{r-4}{2}\times\frac{r-5}{3}x^{r-6}$ — &c; ubi terminorum numerus semper est par, a diametri quadrato auferatur, & residui radix quadratica extrahatur, posito $\lambda=\frac{1}{r}$, erit illa radix $=\pm x^1\mp\lambda x^{\lambda-2}\pm\lambda\times\frac{\lambda-3}{2}x^{\lambda-4}\mp$ &c. ut antea. Ideoque quando arcus numero pari multiplicatur, hæc noviſſima ſeries chordam exhibebit complementi ad ſemicirculum arcus multiplicis, ſi ita ſumantur tot illius termini, quot ſunt unitates in $\frac{1}{2}\lambda+1$, ut ultimus terminus ſit affirmativus *.

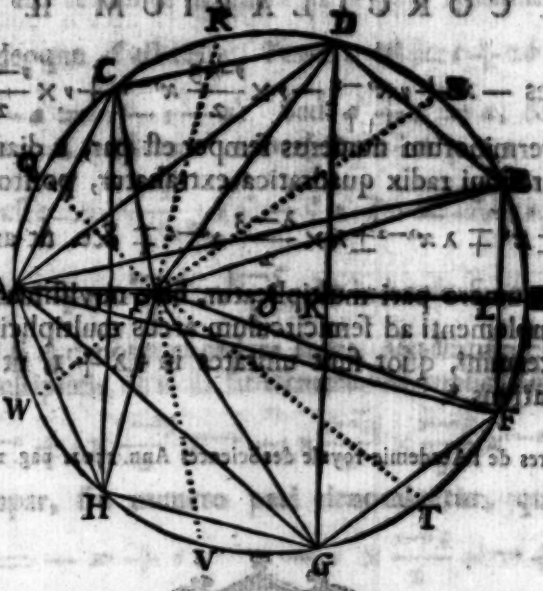
* Vid. Memoires de l'Academie royale des Sciences Ann. 1702. pag. 285.

PROPO-

PROPOSITIO III.

Invenire latus cujusvis polygoni regularis in circulo inscripti.

In circulo AB describatur polygonum, cujus unum ex lateribus sit AC. Quoniam propositione præcedenti si AC sit $= x$, &c



AC aliquoties multiplicetur, item si quadrans arcus multiplicatur, tot unitates sint in $\frac{1}{4}\nu$, quadrantum chordis talis arcus multiplicis erit $= x^{\nu} - \nu x^{\nu-2} + \nu \times \frac{\nu-3}{2} x^{\nu-4} - \nu \times \frac{\nu-4}{2} \times \frac{\nu-5}{3} x^{\nu-6} +$ &c; aut $= -x^{\nu} + \nu x^{\nu-2} - \nu \times \frac{\nu-3}{2} x^{\nu-4} +$ &c, terminis harum serierum sumptis numero æqualibus $\frac{1}{4}\nu$. Si igitur arcus multiplex sit totus circulus, ejus chorda est $= 0$; & chorda AC hac æquatione determinabitur $0 = x^{\nu} - \nu x^{\nu-2} + \nu \times \frac{\nu-3}{2} x^{\nu-4} - \nu \times \frac{\nu-4}{2} \times \frac{\nu-5}{3} x^{\nu-6} +$ &c; terminis hujus seriei sumptis numero æqualibus $\frac{1}{4}\nu$. Sed numerus, quo ν in indice singuli termini post primum diminuitur, æqualis semper est duplo

duplo numero terminorum præcedentium; cum igitur terminorum numerus ſumptus ſit $= \frac{1}{2}\nu$, potestas x in termino ultimo erit ejus quadratum. Unde æquatio hoc quadrato dividi poteſt, & inde fiet $0 = x^{\nu-1} - \nu x^{\nu-4} + \nu \times \frac{\nu-3}{2} x^{\nu-7} - \nu \times \frac{\nu-4}{2} \times \frac{\nu-5}{3} x^{\nu-10} + \&c.$

Deinde obſervare oportet hanc æquationem non tantum habere pro radice latus polygoni, quinetiam lineas jungentes angulos quoſvis duos polygoni: nam, cum omnis talis linea arcum ſubtendit, qui aliquoties multiplex eſt arcus AC, ſi propoſitione præcedenti æquationem formes inter illius lineæ quadratum & quantitatem ex poteſtatibus AC compoſitam, quæ in iſta propoſitione hujus lineæ quadrato æqualis demonſtrata fuit; & hoc pacto ex æquatione extermines AC vel x, & in ejus locum hanc alteram chordam ſubſtituas, ſemper produces vel eandem æquationem, vel aliam, qua hæc, ſcripto prius novæ hujus chordæ quadrato loco xx, dividi poteſt.

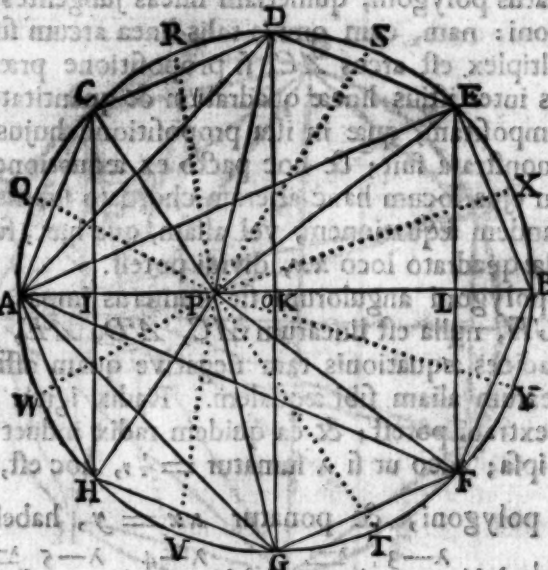
Jam ſi polygoni angulorum ſit numerus impar, ut in polygono $ACDEFGH$, nulla eſt linearum AC, AD, AE, AF, AG, AH, quæ ſunt radices æquationis tam negative quam affirmative ſumptæ, ſed habet earum aliam ſibi æqualem. Radix igitur quadratica hujus æquationis extrahi poteſt; & ea quidem radix induet eandem formam ac æquatio ipſa; adeo ut ſi λ ſumatur $= \frac{1}{2}\nu$, hoc eſt, æqualis numero angulorum polygoni, & ponatur $xx = y$, habebimus $0 = y^{\frac{\lambda-1}{2}} - \lambda y^{\frac{\lambda-3}{2}} + \lambda \times \frac{\lambda-3}{2} y^{\frac{\lambda-5}{2}} - \lambda \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} y^{\frac{\lambda-7}{2}} + \&c$; cujus æquationis radices ſunt linearum AC, AD, AE quadratæ.

Porro ſi polygoni angulorum ſit numerus par, ut in polygono $ACDEBFGH$[1], linearum AC, AD, AE, AB, AF, AG, AH unaquæque habet quoque aliam ex eis ſibi æqualem, præter diametrum ipſam AB, quæ eſt æquationis etiam radix; & cum ſemidiameter loco unitatis habeatur, poteſt æquatio per $xx - 4$ dividi, & radix quadratica quotientis extrahi; quibus rebus peractis, ſi λ ſumatur $= \frac{1}{2}\nu$, hoc eſt, angulorum polygoni numero æqualis, & $xx = y$, hanc æquationem habebimus $0 = y^{\frac{1}{2}\lambda-1} - \sqrt{\lambda-2} \times y^{\frac{1}{2}\lambda-2} + \overline{\lambda-3} \times \frac{\lambda-4}{2} y^{\frac{1}{2}\lambda-3} - \sqrt{\lambda-4} \times \frac{\lambda-5}{2} \times \frac{\lambda-6}{3} y^{\frac{1}{2}\lambda-4} + \&c$, cujus radices ſunt ACq, ADq, AEq.

[1] Vid. Fig. pag. ſequenti

COROLLARIUM I.

SI ex angulis polygoni C, D, E perpendiculares CI, DK, EL in circuli diametrum AB demittantur, quia ACq est $= 2AI \times$ semidiametrum, $ADq = 2AK \times$ semidiametrum circuli, &c, si ista semidiameter pro unitate sit sumpta; duplum uniuscujusque linearum



AI, AK, AL, radix quoque erit æquationum præcedentium; nimirum cum polygoni angulorum est numerus impar, hujus æquationis $0 = y^{\frac{\lambda-1}{2}} - \lambda y^{\frac{\lambda-3}{2}} + \lambda \times \frac{\lambda-3}{2} y^{\frac{\lambda-5}{2}} - \lambda \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} y^{\frac{\lambda-7}{2}} +$ &c, sin polygoni angulorum sit numerus par, æquationis hujus $0 = y^{\lambda-1} - |\overline{\lambda-2} \times y^{\lambda-2} + \overline{\lambda-3} \times \frac{\lambda-4}{2} y^{\lambda-3} - |\overline{\lambda-4} \times \frac{\lambda-5}{2} \times \frac{\lambda-6}{3} y^{\lambda-4} +$ &c.

COROLLARIUM II.

SI O sit circuli centrum, quoniam in omni æquatione coefficiens termini secundi, signo ejus mutato, æqualis est aggregato omnium radicum; coefficiens termini tertii æqualis omnibus productis ex multiplicatione

plicatione binarum ſingularum radicum oriundis; coefficiens termini quarti, ſigno mutato, æqualis omnibus productis ex multiplicatione ternarum ſingularum radicum, & ita de cæteris; $2AI + 2AK + 2AL$ erit $= \lambda$, hoc eſt $= \lambda \times AO$ (AO exiſtente $= 1$) in primo caſu, ſed in caſu altero $= \lambda - 2 = \overline{\lambda - 2} \times AO$; $2AI \times 2AK + 2AI \times 2AL + 2AK \times 2AL = \lambda \times \frac{\lambda - 3}{2} = \lambda \times \frac{\lambda - 3}{2} \times AOq$ in primo caſu, ſed in altero caſu $= \overline{\lambda - 3} \times \frac{\lambda - 4}{2} = \overline{\lambda - 3} \times \frac{\lambda - 4}{2} \times AOq$; & $2AI \times 2AK \times 2AL = \lambda \times \frac{\lambda - 4}{2} \times \frac{\lambda - 5}{3} = \lambda \times \frac{\lambda - 4}{2} \times \frac{\lambda - 5}{3} \times AOc$ in primo caſu, ſed in caſu altero $= \overline{\lambda - 4} \times \frac{\lambda - 5}{2} \times \frac{\lambda - 6}{3} = \overline{\lambda - 4} \times \frac{\lambda - 5}{2} \times \frac{\lambda - 6}{3} \times AOc$.

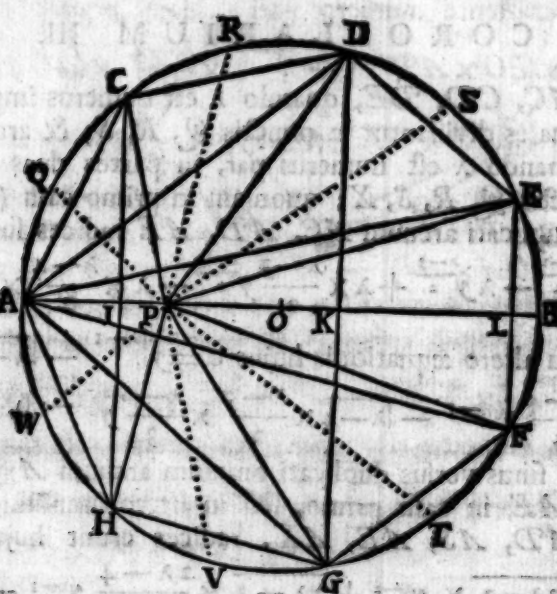
COROLLARIUM III.

SI arcus AC, CD, DE, quando λ eſt numerus impar, in partes duas æquales dividantur in punctis Q, R, S; & arcus AC, CD, DE, EB, quando λ eſt numerus par, in partes duas æquales dividantur in punctis Q, R, S, X; quoniam in primo caſu (per *Corol.* 1.) ſinus verſus duplicati arcuum AC, AD, AE radices ſunt æquationis hujus $o = y^{\frac{\lambda-1}{2}} - \lambda y^{\frac{\lambda-3}{2}} + \lambda \times \frac{\lambda-3}{2} y^{\frac{\lambda-5}{2}} - \lambda \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} y^{\frac{\lambda-7}{2}} +$ &c; & in caſu altero æquationis hujus $o = y^{\frac{1}{2}\lambda-1} - \overline{\lambda-2} \times y^{\frac{1}{2}\lambda-2} + \overline{\lambda-3} \times \frac{\lambda-4}{2} y^{\frac{1}{2}\lambda-3} - \overline{\lambda-4} \times \frac{\lambda-5}{2} \times \frac{\lambda-6}{3} y^{\frac{1}{2}\lambda-4} +$ &c; item (per idem *Coroll.*) ſinus verſus duplicati omnium arcuum AQ, AC, AR, AD, AS, AE in caſu primo, ſed in altero omnium arcuum AQ, AC, AR, AD, AS, AE, AX, radices erunt hujus æquationis $o = y^{\lambda-1} - \overline{2\lambda-2} \times y^{\lambda-2} + \overline{2\lambda-3} \times \frac{2\lambda-4}{2} y^{\lambda-3} - \overline{2\lambda-4} \times \frac{2\lambda-5}{2} \times \frac{2\lambda-6}{3} y^{\lambda-4} +$ &c; ſi æquatio hæc poſtrema per priores dividatur, æquationes inveniemus, quarum radices erunt ſinus verſus duplicati.

duplicati arcuum AQ, AR, AS, quando λ numerus eſt impar, & arcuum AQ, AR, AS, AX, quando λ eſt numerus par: erit autem illarum æquationum prior $0 = y^{\frac{\lambda-1}{2}} - \overline{\lambda-2} \times y^{\frac{\lambda-3}{2}} + \overline{\lambda-3} \times \frac{\lambda-4}{2} y^{\frac{\lambda-5}{2}} - \overline{\lambda-4} \times \frac{\lambda-5}{2} \times \frac{\lambda-6}{3} y^{\frac{\lambda-7}{2}} +$ &c, poſterior autem $0 = y^{\lambda} - \lambda y^{\frac{\lambda}{2}-1} + \lambda \times \frac{\lambda-3}{2} y^{\frac{\lambda}{2}-2} - \lambda \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} y^{\frac{\lambda}{2}-3} +$ &c.

COROLLARIUM IV.

SUMMA ſinuum verſorum duplicatorum arcuum AQ, AR, AS, quando λ eſt numerus impar, erit $= \overline{\lambda-2} \times AO$, & ſumma ſinuum verſorum duplicatorum arcuum AQ, AR, AS, AX, quando λ numerus eſt par, erit $= \lambda \times AO$; ſumma rectangulorum ſub ſingulis



binis e ſinibus his verſis duplicatis erit in caſu priori $= \overline{\lambda-3} \times \frac{\lambda-4}{2} \times AOq$, &c in caſu altero $= \lambda \times \frac{\lambda-3}{2} AOq$; item ſumma omnium

ſolidorum

ſolidorum ſub ſingulis terminis e ſinubus his verſis duplicatis vel $=\overline{\lambda-4}\times\frac{\lambda-5}{2}\times\frac{\lambda-6}{3}\times AOc$, vel $=\lambda\times\frac{\lambda-4}{2}\times\frac{\lambda-5}{3}\times AOc$; & ſimili ratione ſe habent cætera harum linearum producta.

PROPOSITIO IV.

SI punctum aliquod P ſumatur in diametro AB, & lineæ PC, PD, PE, PF, PG, PH ducantur, dico, λ denotante numerum angulorum polygoni, $PC\times PD\times PE\times PF\times PG\times PH$ æquale eſſe

$$AP^{\lambda-1}+\lambda\times AO\times PO\times AP^{\lambda-2}+\lambda\times\frac{\lambda-3}{2}AOq\times POq\times AP^{\lambda-3}$$

$$+\lambda\times\frac{\lambda-4}{2}\times\frac{\lambda-5}{3}AOc\times POc\times AP^{\lambda-4}+\&c,$$

cum λ ſit numerus impar; ſed λ exiſtente numero pari, idem æquale eſſe $AP^{\lambda-1}$

$$+\overline{\lambda-2}\times AO\times PO\times AP^{\lambda-3}+\overline{\lambda-3}\times\frac{\lambda-4}{2}\times AOq\times POq\times$$

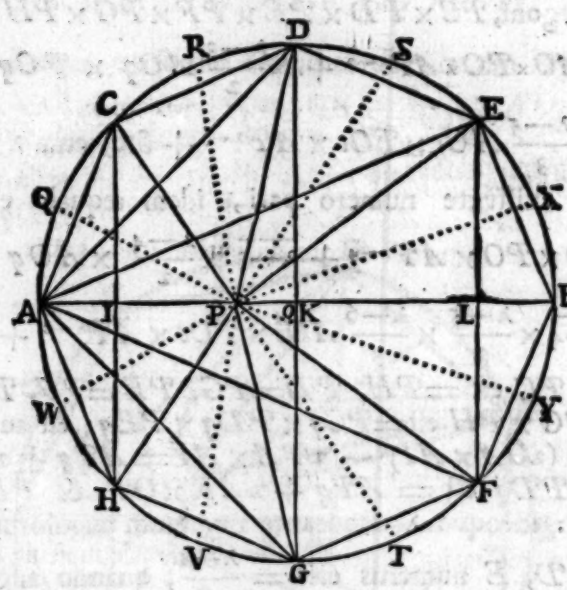
$$AP^{\lambda-4}+\overline{\lambda-4}\times\frac{\lambda-5}{2}\times\frac{\lambda-6}{3}AOc\times POc\times AP^{\lambda-5}+\&c.$$

QUONIAM PC eſt $=PH$, $PD=PG$, $PE=PF$, $PC\times PD\times PE\times PF\times PG\times PH$ eſt $=PCq\times PDq\times PEq$; eſt autem $PCq=APq+ACq\ (2OA\times AI)-2PA\times AI=APq+2AI\times OP$; eodem modo PDq eſt $=APq+2AK\times OP$, & $PEq=APq+2AL\times OP$: ideoque λ denotante numerum angulorum polygoni, angulorum C, D, E numerus erit $=\frac{\lambda-1}{2}$, quando angulorum polygoni omnium numerus eſt impar; ſed quando eſt numerus eorum par, numerus angulorum C, D, E, erit $=\frac{\lambda-2}{2}$; erit igitur $PCq\times PDq\times PEq\ (=PC\times PD\times PE\times PF\times PG\times PH)=AP^{\lambda-1}+\overline{2AI+2AK+2AL}\times PA^{\lambda-2}\times OP+\overline{2AI\times 2AK+2AI\times 2AL+2AK\times 2AL}\times PA^{\lambda-3}\times OPq+2AI\times 2AK\times 2AL\times AP^{\lambda-4}\times OPc$, ſi polygonum angulos habet numero impares; ſed cum numero ſunt pares $PCq\times PDq\times PEq$ erit $=AP^{\lambda-1}+\overline{2AI+2AK+2AL}\times AP^{\lambda-2}\times OP+\overline{2AI\times 2AK+2AI\times 2AL+2AK\times 2AL}\times AP^{\lambda-3}\times OPq+2AI\times 2AK\times 2AL\times$

$AP^{\lambda-3} \times OPc$. Unde apparet propositionis veritas *corollario secundo præcedentis.*

PROPOSITIO V.

ISDEM positis, si angulorum numerus sit impar, dico $AP \times PC \times PD \times PE \times PF \times PG \times PH$ esse $= AO^{\lambda} - OP^{\lambda}$; sin angulorum sit numerus par, tum $AP \times PC \times PD \times PE \times PB \times PF \times PG \times PH$ fore $= AO^{\lambda} - OP^{\lambda}$.



QUONIAM AP est $= AO - OP$, in casu primo $AP \times PC \times PD \times PE \times PF \times PG \times PH$ erit $= \overline{AO-OP} \times \overline{AO-OP}^{\lambda-1} + \lambda \times AO \times PO \times \overline{AO-OP}^{\lambda-3} + \lambda \times \frac{\lambda-3}{2} \times AOq \times POq \times \overline{AO-OP}^{\lambda-5} + \lambda \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} AOc \times OPc \times \overline{AO-OP}^{\lambda-7}$; & in casu secundo, cum $AP \times PB$ est $= AOq - OPq$, $AP \times PC \times PD \times PE \times PB \times PF \times PG \times PH$ erit $= \overline{AOq-OPq} \times \overline{AO-OP}^{\lambda-2} + \lambda-2 \times AO \times PO \times \overline{AO-OP}^{\lambda-4} + \overline{\lambda-3} \times \frac{\lambda-4}{2} AOq \times POq \times AO-OP$

$\overline{AO-OP}|^{\lambda-6}+\overline{\lambda-4}\times\frac{\lambda-5}{2}\times\frac{\lambda-6}{3}AOc\times POc\times\overline{AO-OP}|^{\lambda-8}$,

Si binomium igitur $AO-OP$ elevetur ad poteſtates ſuas ex magni *Newtoni* præceptis, in primo caſu $\overline{AP\times PC\times PD\times PE\times PF\times PG\times PH}$ invenietur $=\overline{AO-OP}\times\overline{AO^{\lambda-1}+AO^{\lambda-2}\times OP+AO^{\lambda-3}\times OPq+AO^{\lambda-4}\times OPc}+$&c, nimirum $=AO^{\lambda}-OP^{\lambda}$; & in caſu ſecundo $\overline{AP\times PC\times PD\times PE\times PB\times PF\times PG\times PH}$ invenietur $=\overline{AOq-OPq}\times\overline{AO^{\lambda-2}+AO^{\lambda-4}\times OPq+AO^{\lambda-6}\times OPqq}+$&c, ſive $=AO^{\lambda}-OP^{\lambda}$.

ALITER.

Cum invenerimus $PCq=APq+2AI\times OP$, $PDq=APq+2AK\times OP$, & $PEq=APq+2AL\times OP$; & æquationum, quas in *Corollario primo Propoſitionis tertiæ* dedimus, radices ſunt omnes ſinus verſus duplicati $2AI$, $2AK$, $2AL$, ſi pro PCq, PDq, PEq promiſcue ponamus v, a prima earum æquationum inveniemus

$$0=v^{\frac{\lambda-1}{2}}-\frac{\lambda-1}{2}APq\times v^{\frac{\lambda-3}{2}}+\frac{\lambda-1}{2}\times\frac{\lambda-3}{4}APqq\times v^{\frac{\lambda-5}{2}}\ldots\ldots\pm AP^{\lambda-1}$$
$$-\lambda\times AO\times OP\times v^{\frac{\lambda-3}{2}}+\lambda\times\frac{\lambda-3}{2}AO\times OP\times APq\times v^{\frac{\lambda-5}{2}}\ldots\pm\lambda\times AO\times OP\times AP^{\lambda-3}$$
$$+\lambda\times\frac{\lambda-3}{2}AOq\times OPq\times v^{\frac{\lambda-5}{2}}\ldots\ldots\pm\lambda\times\frac{\lambda-3}{2}AOq\times OPq\times AP^{\lambda-5}$$
&c.

ab altera vero æquatione in illo *corollario* deducitur hæc

$$0=v^{\frac{\lambda-2}{2}}-\overline{\frac{\lambda-2}{2}}\times APq\times v^{\frac{\lambda-4}{2}}+\frac{\lambda-2}{2}\times\frac{\lambda-4}{4}\times APqq\times v^{\frac{\lambda-6}{2}}\ldots\pm AP^{\lambda-2}$$
$$-\overline{\lambda-2}\times AO\times OP\times v^{\frac{\lambda-4}{2}}+\overline{\lambda-2}\times\frac{\lambda-4}{2}AO\times OP\times APq\times v^{\frac{\lambda-6}{2}}\ldots\pm\overline{\lambda-2}\times AO\times OP\times AP^{\lambda-4}$$
$$+\overline{\lambda-3}\times\frac{\lambda-4}{2}\times AOq\times OPq\times v^{\frac{\lambda-6}{2}}\ldots\pm\overline{\lambda-3}\times\frac{\lambda-4}{2}AOq\times OPq\times AP^{\lambda-6}$$
&c.

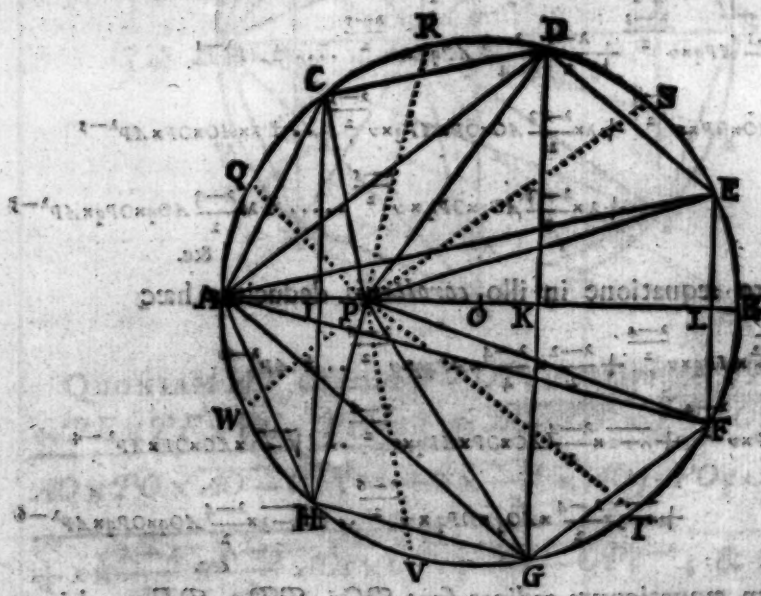
Harum autem æquationum radices ſunt PCq, PDq, PEq; ultimus autem terminus in utriſque affirmativus eſt, quando numerus terminorum in ſuprema æquationis ſerie eſt impar, & negativus quando iſtorum terminorum numerus eſt par. Ideoque productum ab omnibus radicibus

radicibus nempe $PCq \times PDq \times PEq$ in priori æquatione, ubi λ eſt numerus impar, ſemper erit $= AP^{\lambda-1} + \lambda \times AO \times OP \times AP^{\lambda-3} + \lambda \times \frac{\lambda-3}{2} \times AOq \times OPq \times AP^{\lambda-5} +$ &c. & in æquatione poſteriori, ubi λ eſt numerus par, idem productum erit $= AP^{\lambda-1} + \overline{\lambda-2} \times AO \times OP \times AP^{\lambda-3} + \overline{\lambda-3} \times \frac{\lambda-4}{2} AOq \times OPq \times AP^{\lambda-5} +$ &c.

Unde oſtenditur ut antea, quando λ numerus eſt impar, eſſe $AO^{\lambda} - OP^{\lambda} = AP \times PCq \times PDq \times PEq$, & quando λ eſt numerus par, eſſe $AO^{\lambda} - OP^{\lambda} = AP \times PB \times PCq \times PDq \times PEq$.

PROPOSITIO VI.

Si in caſu primo arcus AC, CD, DE, FG, GH, HA dividantur in duas partes æquales in punctis Q, R, S, T, V, W; & in caſu ſecundo arcus EB, BF etiam dividantur in duas partes æquales in X, Y, & lineæ $PQ, PW, PR, PV, PS, PT, PX, PY$ ducantur,



dico, in caſu primo $PQ \times PR \times PS \times PB \times PT \times PV \times PW$ eſſe $= AO^{\lambda} + OP^{\lambda}$: & in altero caſu $PQ \times PR \times PS \times PX \times PT \times PY \times PV \times PW$ eſſe $= AO^{\lambda} + OP^{\lambda}$.

In

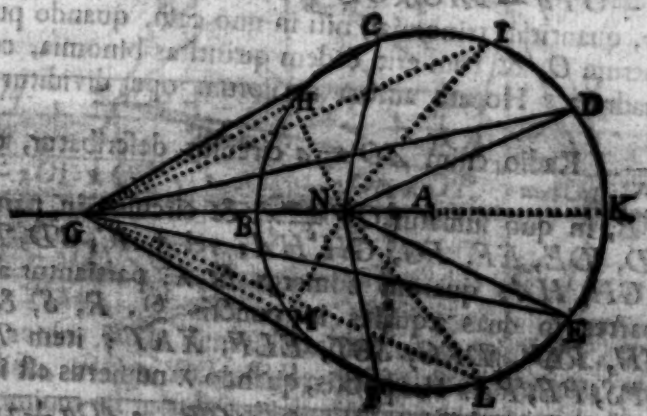
In casu primo *propositione novissima* $AP \times PC \times PD \times PE \times PF \times PG \times PH$ est $= AO^{\lambda} - OP^{\lambda}$; & *eadem propositione* $AP \times PQ \times PC \times PR \times PD \times PS \times PE \times PB \times PF \times PT \times PG \times PV \times PH \times PW$ est $= AO^{2\lambda} - OP^{2\lambda}$: ideoque $PQ \times PR \times PS \times PB \times PT \times PV \times PW$ erit $= \frac{AO^{2\lambda} - OP^{2\lambda}}{AO^{\lambda} - OP^{\lambda}} = AO^{\lambda} + OP^{\lambda}$.

In casu altero $AP \times PC \times PD \times PE \times PB \times PF \times PG \times PH$ est $= AO^{\lambda} - OP^{\lambda}$, & $AP \times PQ \times PC \times PR \times PD \times PS \times PE \times PX \times PB \times PT \times PF \times PT \times PG \times PV \times PH \times PW$ est $= AO^{2\lambda} - OP^{2\lambda}$: unde $PQ \times PR \times PS \times PX \times PT \times PT \times PV \times PW$ est $= \frac{AO^{2\lambda} - OP^{2\lambda}}{AO^{\lambda} - OP^{\lambda}} = AO^{\lambda} + OP^{\lambda}$.

Idem quoque demonstrari potest *coroll. quart.* vel *tert. prop. tert.*

PROPOSITIO VII.

Postremo, si circumferentia, cujus centrum est *A*, divisa sit in quemvis numerum partium æqualium in punctis *B*, *C*, *D*, *E*, *F*, quem numerum denotet λ, si in semidiametro producta *BA* punctum *G* sumatur extra circulum, & ducantur lineæ *GC*, *GD*, *GE*, *GF*, dico $GB \times GC \times GD \times GE \times GF$ fore $= GA^{\lambda} - BA^{\lambda}$. Et præterea, si singuli arcus *BC*, *CD*, *DE*, *EF*, *FB* divisi sint in duas partes æquales in punctis *H*, *I*, *K*, *L*, *M*, lineis *GH*, *GI*, *GK*, *GL*, *GM* ductis, dico $GH \times GI \times GK \times GL \times GM$ fore $= GA^{\lambda} + BA^{\lambda}$.



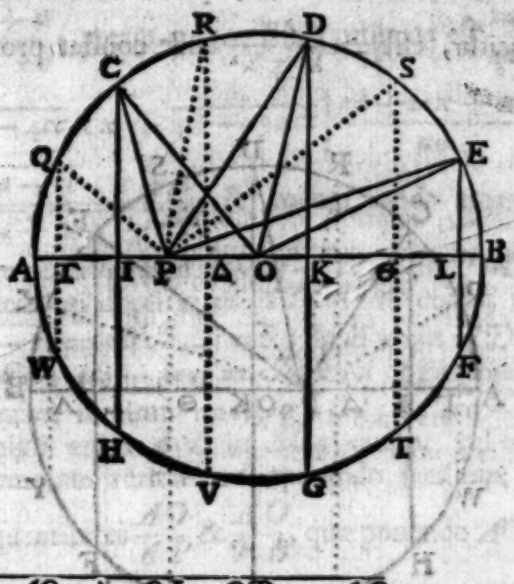
Si in AB punctum N ſumatur, ita ut AN ſit tertia proportionalis ad GA & BA, & lineæ NH, NC, NI, ND, NE, NL, NF, NM ducantur, propoſitione librorum deperditorum *Apollonii de locis planis*, cujus demonſtrationem nobis conſervavit *Eutocius* *, eſt ut $GA : AB$ $(:: AB : AN) :: GB : BN :: GH : HN :: GC : CN :: GI : IN :: GD : DN :: GK : KN :: GE : EN :: GL : LN :: GF : FN :: GM : MN$. Igitur cum $BN \times NC \times ND \times NE \times NF$ ſit $= AB^{\lambda} - AN^{\lambda}$, $GB \times GC \times GD \times GE \times GF$ erit $= GA^{\lambda} - AB^{\lambda}$. Et cum $NH \times NI \times NK \times NL \times NM$ ſit $= AB^{\lambda} + AN^{\lambda}$, $GH \times GI \times GK \times GL \times GM$ erit $= GA^{\lambda} + AB^{\lambda}$.

Jam his demonſtratis ad aliud auctoris noſtri egregium inventum tranſeundum eſt, & oſtendendum quomodo fractio $\frac{1}{a^{\lambda} \mp x^{\lambda}}$ in alias fractiones ſit dividenda, quarum denominatoribus binomia & trinomia ſimplicia tribuantur. Manifeſtum autem eſt binomium $a^{\lambda} \mp x^{\lambda}$ in binomios & trinomios factores ſimplices dividi *propoſitionum duarum proxime præcedentium* ope; nam ſumendo $AO = a$, & $OP = x$, ſegmenta diametri circuli binomia ſunt exiſtente $AP = a - x$, & $BP = a + x$. Et cum omnes reliqui diviſores per binos ſint æquales, uniuſcujuſque quadratum eſt etiam diviſor; hæc autem quadrata ſunt plerumque trinomia. Verbi cauſa PCq $(= APq + 2AI \times OP)$ eſt $= AOq + OPq - 2AO \times OP + 2AI \times OP = AOq - 2OI \times OP + OPq$, quantitati trinomiæ, niſi in uno caſu, quando punctum I cadit in centrum O, & tum erit eadem quantitas binomia, cujus partes ſunt quadrata. Horum autem diviſorum ope dividitur quoque fractio $\frac{1}{a^{\lambda} \mp x^{\lambda}}$. Radio enim $AO = a$ circulus deſcribatur, cujus diameter ſit AB, in quo ſumatur $OP = x$, & circulus in partes æquales AC, CD, DE, EF, FG, GH, HA; vel † AC, CD, DE, EB, BF, BG, GH, HA; quarum numerus ſit λ; partiantur autem ſingulæ hæ partes in duas æquales in punctis Q, R, S, &c, ductis QTW, CIH, $R\Delta V$, DKG, $S\Theta T$, ELF, $X\Lambda Y$; item PQ, PC, PR, PD, PS, PE, PX. His poſitis, quando λ numerus eſt impar, erit

$$\frac{1}{a^{\lambda} - x^{\lambda}} = \frac{1}{\lambda a^{\lambda}} \times \frac{AO}{AP} + \frac{2AOq - 2OI \times OP}{PCq} + \frac{2AOq + 2OK \times OP}{PDq} + \overline{2AO}$$

* Commen. in lib. 1. Conic. Apollon. † Vid. fig. in pag. 30.

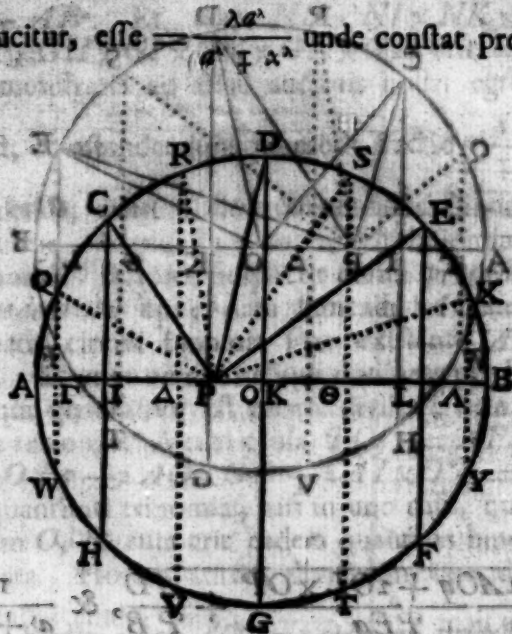
$+\frac{2AOq+2OL\times OP}{PEq}$, & $\frac{1}{a^2+x^2}=\frac{1}{\lambda a^2}\times\frac{2AOq-2O\Gamma\times OP}{PQq}$
$+\frac{2AOq-2O\Delta\times OP}{PRq}+\frac{2AOq+2O\Theta\times OP}{PSq}+\frac{AO}{PB}$; quando autem λ est numerus par, erit $\frac{1}{a^2-x^2}=\frac{1}{\lambda a^2}\times\frac{AO}{AP}+\frac{2AOq-2OI\times OP}{PCq}$



$+\frac{2AOq}{PDq}+\frac{2AOq+2OL\times OP}{PEq}+\frac{AO}{PB}$, & $\frac{1}{a^2+x^2}=\frac{1}{\lambda a^2}\times$
$\frac{2AOq-2O\Gamma\times OP}{PQq}+\frac{2AOq-2O\Delta\times OP}{PRq}+\frac{2AOq+2O\Theta\times OP}{PSq}$
$+\frac{2AOq+2O\Lambda\times OP}{PXq}$.

Hæc autem facile demonstrantur; cum enim AP, PB binomia sint simplicia, PQq, PCq, PRq, PDq, &c. trinomia præter PDq, quod binomium est membris quadratis; pro his ponendo binomia & trinomia sua, & deinde reducendo omnes terminos ductos in $\frac{1}{\lambda a^2}$ ad communem denominatorem, primus numeratoris terminus invenietur æqualis primo denominatoris ducto in λ, secundus numeratoris terminus secundo

secundo denominatoris æqualis erit ductio in $\lambda - z$, & tertius terminus numeratoris tertio denominatoris æqualis ducto in $\lambda - z$, legemque similem omnes numeratoris termini observant; & termino aliquo in numeratore deficiente terminus correspondens in denominatore etiam deficit. Imo cum demonstraverimus hunc denominatorem esse $= a^2 \mp x^2$, ita ut omnes ejus termini præter primum & ultimum evanescant, omnes numeratoris termini præter primum evanescent; ille autem primus terminus erit $= \lambda a^2$, quod ostendit totam quantitatem, quæ in $\frac{1}{\lambda a^2}$ ducitur, esse $= \frac{\lambda a^2}{a^2 \mp x^2}$ unde constat propositum.



$$\times \frac{1}{\lambda a^2} = \frac{1}{a^2 \mp x^2} \&c. + \frac{1}{\quad} + \frac{1}{\quad} +$$

Hac ratione demonstrantur principia, in quæ auctor noster illustris curvarum extruxit quadraturam, quarum ordinatæ denominatores habent binomios & trinomios. Ut autem hinc curvas illas metiamur præterea observandum est, quod si ex quantitatibus præcedentibus in $\frac{1}{\lambda a^2}$ ductis aliquam sumas, quæ trinomium habet divisorem, puta $\frac{2AOq + 2OK \times OP}{PDq}$, & si ponas $AO = a$, $OK = c$, & $OP = x$; ut sit $\frac{2AOq + 2OK \times OP}{PDq} = \frac{2aa + 2cx}{aa + xx + xx}$; tabula decima tertia seriei tabularum prioris in *Cotesii* opere invenies aream curvæ, cujus abscissa

abſciſſa eſt x & ordinata $\frac{2aa+2cx}{aa+2cx+xx}$, $= c\left|\frac{aa+2cx+xx}{aa}\right.$ $+2\sqrt{aa-cc}\left(\frac{\sqrt{aa-cc}\,.\,c+x}{\sqrt{aa+2cx+xx}}\right.$ *; aream curvæ, cujus ordinata eſt $\frac{2aax+2cxx}{aa+2cx+xx}$, $=2cx+\sqrt{aa-2cc}\left|\frac{aa+2cx+xx}{aa}\right.$ $+4c\sqrt{aa-cc}\left(\frac{\sqrt{aa-cc}\,.\,c+x}{\sqrt{aa+2cx+xx}}\right.$; aream curvæ, cujus ordinata eſt $\frac{2aaxx+2cx^3}{aa+2cx+xx}$, $=cx^2+\sqrt{2aa-4cc}\times x+4c^3-3aac\left|\frac{aa+2cx+xx}{aa}\right.$ $+8cc-2aa\sqrt{aa-cc}\left(\frac{\sqrt{aa-cc}\,.\,c+x}{\sqrt{aa+2cx+xx}}\right.$; & eodem modo areas cæteras invenies. Hic autem obſervandum eſt $\sqrt{aa-cc}$ = eſſe DK; $2cc-aa$ = eſſe ſinui complementi dupli arcus AD in AO ducto, nempe = $AO\times OL$; $2c\sqrt{aa-cc}$ = eſſe ſinui recto dupli arcus AD in AO ducto, id eſt = $AO\times FL$; $4c^3-3aac$ = eſſe ſinui complementi tripli arcus AD in AOq ducto, nimirum = $AOq\times OI$; & $4cc-aa\sqrt{aa-cc}$ = eſſe ſinui tripli arcus AD in AOq ducto, vel = $AOq\times HI$. Horum autem omnium veritatem ſupputando facillime experiri potes.

Porro ſi ſumas quantitates $\frac{AO}{AP}$ & $\frac{AO}{PB}$, quæ ponendo $AO=a$, $OP=x$, ut antea, æquales fient $\frac{a}{a-x}$, $\frac{a}{a+x}$, prima tabula ſerici prioris invenies aream curvæ, cujus ordinata eſt $\frac{a}{a-x}$, $=-a\left|\frac{a-x}{a}\right.$; aream curvæ, cujus ordinata eſt $\frac{ax}{a-x}$, $=-ax-aa\left|\frac{a-x}{a}\right.$; aream curvæ, cujus ordinata eſt $\frac{ax^2}{a-x}$, $=-\frac{1}{2}axx-aax-a^3\left|\frac{a-x}{a}\right.$: aream autem curvæ, cujus ordinata eſt $\frac{a}{a+x}$, $=a\left|\frac{a+x}{a}\right.$; aream curvæ, cujus ordinata eſt $\frac{ax}{a+x}$, $=ax-aa\left|\frac{a+x}{a}\right.$; & aream curvæ, cujus

* Conſer Tab. hanc 13. cum Not. in Harmon. Menſur. pag. 98.

ordinata est $\frac{axx}{a+x}$, $= \frac{1}{2}axx - aax + a^3 \left| \frac{a+x}{a} \right.$; & eodem modo aræ cæteræ invenientur.

JAM vero hinc intelligitur ratio inveniendi omnium curvarum memoratarum areas, quarum ordinatæ hanc habent formam $\frac{dz^{\frac{\delta}{\lambda}-1}}{e+fz^{\frac{\delta}{\lambda}}}$, δ & λ numeros quoslibet integros denotantibus. Ponendo enim $z^{\frac{\delta}{\lambda}} = x^{\gamma}$, curva, cujus abscissa est z & ordinata jam memorata, æqualis invenietur aræ curvæ, cujus abscissa est x & ordinata $\frac{\frac{\gamma}{\lambda} dx^{\gamma-1}}{e+fx^{\gamma}}$. Sit verbi causa curvæ ordinata $\frac{dz^{\frac{7}{2}-1}}{e+fz^{\frac{7}{2}}}$; hujus curvæ area, ponendo $z^{\frac{7}{2}} = x^7$, æqualis invenietur areæ curvæ, cujus abscissa est x & ordinata $\frac{\frac{7}{2}dx^6}{e+fx^7}$; quæ ordinata, ponendo $\frac{e}{f} = a^7$, fiet $= \frac{\frac{7}{2}xx}{a^7+x^7}$; & area huic posteriori ordinatæ respondens ex iis, quæ jam tradidi, facilem mensuram habet. Circulum enim describendo radio $AO = a$, illum dividendo in partes septem æquales in punctis A, C, D, E, F, G, H, sumendoque $OP = x$, & ducendo PC, PD, PE, erit ordinata

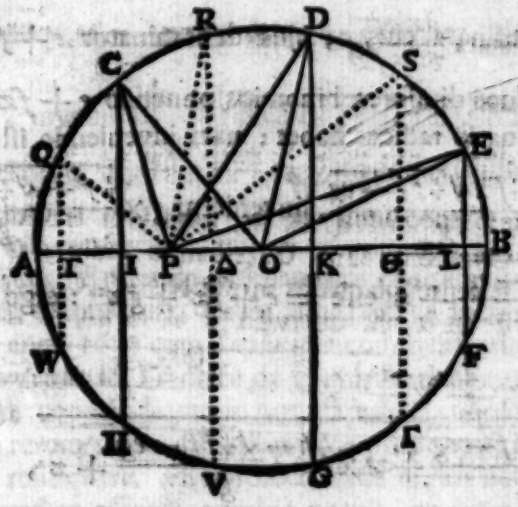
$$\frac{\frac{7d}{2f}xx}{a^7-x^7} = \frac{7d}{2f} \times \frac{1}{7a^7} \times \frac{AO \times x^2}{AP} + \frac{2AOq \times x^2 - 2OI \times x^3}{PCq} + \frac{2AOq \times x^2 + 2OK \times x^3}{PDq} + \frac{2AOq \times x^2 + 2OL \times x^3}{PEq}.$$

Unde, quantitatum signis rite observatis, ex præcedentibus aream invenies æqualem

$$\frac{7d}{2f} \times \frac{1}{7a^7} \times -\frac{1}{4}AO \times xx + AOq \times x - AOc \left| \frac{AP}{AO} - OI \times xx + 2OK \times AO \times x \right.$$
$$+ PL \times AOq \left| \frac{PCq}{AOq} + 2EL \times AOq \left(\frac{CI \cdot IP}{CP} + OK \times xx + 2OL \right. \right.$$
$$\times AO \times x - OI \times AOq \left| \frac{PDq}{AOq} - 2HI \times AOq \left(\frac{DK \cdot PK}{PD} + OL \right. \right.$$
$$\times xx - 2OI \times AO \times x + OK \times AOq \left| \frac{PEq}{AOq} + 2DK \left(\frac{EL \cdot PL}{PE} \right. \right.$$

Demonstra-

Demonſtravimus autem ſupra $2AI + 2AK + 2AL =$ eſſe $7AO$ *; ideoque $2OK + 2OL - 2OI = 2AK - 2AO + 2AL - 2AO + 2AI - 2AO =$ eſt $2AK + 2AL + 2AI - 6AO = AO$; unde erit $-\frac{1}{7}AO \times xx - OI \times xx + OK \times xx + OL \times xx = 0$; item $-AOq \times x + 2OK \times AO \times x + 2OL \times AO \times x - 2OI \times AO \times x = 0$. Præterea area hyperbolica, vel quantitas ei analoga, quam auctor noſter menſuram appellat, ad modulum quemvis formatam, rationis, quam lineæ alicujus quadratum habet ad alius quadratum, adæquat id, quod menſuram ad duplum modulum formatam vocat rationis prioris lineæ ad poſteriorem. Si igitur non ſumas arcus angulis ſub PCI, ſub PDK & ſub PEL ſubtendentes, ſed eorum loco arcus ſumas, qui



angulis ſub PCO, ſub PDO & ſub PEO ſubtendunt, nimirum ut area quæſita ad principium abſciſſæ terminetur; & deinde ſcribendi rationem ſequaris, quæ in hac parte poſteriori inventorum *Coteſii* adhibetur, curvæ, cujus ordinata eſt $\frac{dx^{+n-1}}{e+fx^n}$, area, quando f eſt quantitas negativa, ductis OC, OD, OE, prodibit $= \frac{7d}{nf} \times - \frac{1}{7a^5} \times - \overline{AO(AP : AO) + 2PL(PC : CO) + 2EL(PCO) - 2OI(PD : DO)}$

* Prop. 3. coroll. 2. pag. 21.

$- 2HI$

$-2HI\,(PDO) + 2OK\,(PE : EO) + 2DK\,(PEO)$. Hæc autem quadratura ad eam, quam *Coteſius* dedit, reducitur ponendo $AO = 1$, quam hic habuimus $= a = \sqrt[7]{\frac{e}{f}}$.

EODEM modo invenire licet ejuſdem curvæ aream, quando f quantitas eſt affirmativa.

HACTENUS conatus ſum breviter tibi exponere viam, qua *Coteſius* curvas dimenſus eſt, quarum ordinatæ habent denominatores rationales & binomios. Curvæ autem, quarum ordinatæ habent denominatores rationales multinomios, eodem modo tractandæ ſunt, reſolvendo denominatores in diviſores binomios & trinomios; verbi cauſa, ſi detur $\frac{dz^{\frac{1}{11}-1}}{e+fz+gz^{11}}$ ordinata curvæ; ejus denominator $e+fz^{\prime}+gz^{11}$ reſolvi poteſt in duos diviſores binomios ponendo $e+fz^{\prime}+gz^{11}=0$, quando hæc æquatio radices habet; nam inveniendo iſtas radices, nimirum $z^{\prime}=\frac{-\frac{1}{2}f\pm\sqrt{\frac{1}{4}ff-eg}}{g}$, invenietur $g\times\frac{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}\times\frac{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}$ eſſe $=e+fz^{\prime}+gz^{11}$: ergo $\frac{dz^{\frac{1}{11}-1}}{e+fz^{\prime}+gz^{11}}$ eſt $=\frac{\frac{d}{g}z^{\frac{1}{11}-1}}{\frac{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}\times\frac{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}}$ ad inveniendam

igitur hujus curvæ quadraturam nihil requiritur, niſi ut reducatur fractio $\frac{1}{\frac{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}\times\frac{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}}$ ad quantitatem hujus formæ $\frac{m}{\frac{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}}+\frac{n}{\frac{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}}{g}+z^{\prime}}$; quod fit reducendo has duas fractiones ad eundem denominatorem cum priori, & numeratorum ſummam unitati æquando, unde invenietur

$m=$

$m = \frac{-g}{2\sqrt{\frac{1}{4}ff-eg}}$, & $n = \frac{g}{2\sqrt{\frac{1}{4}ff-eg}}$, ideoque ordinata

$$\frac{dz^{\frac{1}{n}-1}}{e+fz^{n}+gz^{2n}} \left(= \frac{\frac{d}{g}z^{\frac{1}{n}-1}}{\frac{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}}{g}+z^{n}\times\frac{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}}{g}+z^{n}}\right)$$

erit $= \frac{g}{2\sqrt{\frac{1}{4}ff-eg}} \times \frac{dz^{\frac{1}{n}-1}}{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}+gz^{n}} - \frac{dz^{\frac{1}{n}-1}}{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}+gz^{n}}$.

Igitur metiendo duas curvas, quarum ordinatæ sunt $\frac{dz^{\frac{1}{n}-1}}{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}+gz^{n}}$ & $\frac{dz^{\frac{1}{n}-1}}{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}+gz^{n}}$, quadratura curvæ, cujus ordinata est $\frac{dz^{\frac{1}{n}-1}}{e+fz^{n}+gz^{2n}}$, invenietur. Et hæc est quadratura curvæ, quam *Newtonus* ante auctorem noſtrum in forma ſexta tabulæ ſuæ dedit.

HINC autem obiter cognoſci poteſt, quam longo tempore *Newtono* nota fuit hæc via conferendi cum circulo & hyperbola curvas ordinatarum rationalium, dividendo ordinatarum denominatores in diviſores binomios; hæc enim forma, quam jam tractavi, locum habet inter eas, quas in literis anno 1676 cum *Leibnitio* communicatis, recenſet *: ſed hanc rem *Newtonus* in Tractatu de curvarum quadratura, propter iſtius libri brevitatem copioſo ſermone non expoſuit. Voluit enim eo libro ſumma tantum rerum capita difficillimaque proponere, & leviſſima quæque lectoribus relinquere. Et profecto ſatis mirari nequeo, quo pacto divini ingenii auctor efficere omnino potuit, ut intra arctos iſtius nobiliſſimi libelli limites copioſiſſimam illam doctrinam uberrimamque complecteretur. Cum ſingularem hujus eximii tractatus brevitatem conſidero, veniunt identidem in animum, quæ *Pappus* ‡ de *Poriſmatis Euclidis* ſcribit. *Hæc*, inquit, *ſubtilem & naturalem habent contemplationem, neceſſariamque & admodum univerſalem, atque iis, qui perſpicere & perveſtigare valent, valde jucundam.* Et mox *poriſmatibus accidit propoſitiones habere contractas, propter res multas vel reconditas omitti ſolitas, ut multi Geometræ ex parte tantum percipiant, eorum, quæ traduntur, maxime neceſſaria haud capientes.*

* Commerc. Epiſtol. Collin. &c. de analyſi promota. pag. 76.

‡ Collection. Mathem. in Præfat. ad lib. 7.

SED, ut ad rem redeamus, hunc locum auxit *Coteſius* ex iis, quæ binomias ordinatas ſpectantia adinvenit; eorum enim ope omnem curvam metitur, cujus ordinata ſub hac forma cadit $\frac{dz^{\frac{\delta}{\lambda}\nu-1}}{e+fz^{\nu}+gz^{2\nu}}$; quando denominator in duos factores binomios dividi poteſt; cum enim hujus curvæ area æqualis ſit differentiæ inter areas duarum curvarum, quarum ordinatæ ſunt $\frac{g}{2\sqrt{\frac{1}{4}ff-eg}}\times\frac{dz^{\frac{\delta}{\lambda}\nu-1}}{\frac{1}{2}f-\sqrt{\frac{1}{4}ff-eg}+gz^{\nu}}$ & $\frac{g}{2\sqrt{\frac{1}{4}ff-eg}}\times\frac{dz^{\frac{\delta}{\lambda}\nu-1}}{\frac{1}{2}f+\sqrt{\frac{1}{4}ff-eg}+gz^{\nu}}$, ut conſtat ex præmiſſis, & quicunque numeri literis δ, λ deſignentur hæ curvæ a *Coteſii* inventis menſuram habent, curva quoque, cui hæ ſunt æquales, ab illis inventis menſuram accipit.

IN his vero, ſi termini e & $gz^{2\nu}$ eodem ſigno afficiuntur, requiritur, ut $\frac{1}{4}ff$ major ſit quam eg; quando igitur eſt $\frac{1}{4}ff<eg$, quo efficitur, ut æquatio; qua factores inveniemus, nullam habeat radicem, aliam viam inſiſtere oportet; tum enim dividendus eſt trinomius denominator in trinomios factores. In hunc finem *Coteſius* rationem excogitavit partiendi omne trinomium in duo alia, quæ a regula nota ad dividendam æquationem biquadraticam in duas quadraticas deducitur; nam trinomium $e+fz^{\nu}+gz^{2\nu}$ haberi poteſt pro quinquinomio, duobus terminis deficientibus. Quoniam igitur ex ea regula ope æquationis $y^6+2qy^4+\overline{qq-4s}\times yy-rr=0$, quinquinomium uno tanto termino deficiente $x^4+qx^2+rx+s=0$ dividi poteſt in duo trinomia, nimirum in $xx+yx+\frac{1}{2}q+\frac{1}{2}yy-\frac{r}{2y}$, & in $xx-yx+\frac{1}{2}q+\frac{1}{2}yy+\frac{r}{2y}$, in caſu noſtro ponendo $q=\frac{f}{g}$, $r=0$, & $s=\frac{e}{g}$, æquatio $y^6+2qy^4+\overline{qq-4s}\times y^2-rr=0$ fiet $y^4+\frac{2f}{g}y^2+\frac{ff-4eg}{gg}=0$, $y=\sqrt{-\frac{f}{g}\pm2\sqrt{\frac{e}{g}}}$, & duo diviſores trinomii erunt aut $\sqrt{g}\times x^2+\sqrt{-\frac{f}{g}+2\sqrt{\frac{e}{g}}}\times x^{\nu}+\sqrt{\frac{e}{g}}\left(=\frac{\sqrt{eg}+\sqrt{-fg+2g\sqrt{eg}}\times x^{\nu}+gx^{2\nu}}{\sqrt{g}}\right.$

& $\sqrt{g}$

$\& \sqrt{g} \times x - \sqrt{-\frac{f}{g} + 2\sqrt{\frac{e}{g}} \times z^{\frac{1}{n}} + \sqrt{\frac{e}{g}}}$ $(= \frac{\sqrt{eg} - \sqrt{-fg + 2g\sqrt{eg} \times z^{\frac{1}{n}} + gz^2}}{\sqrt{g}})$

aut $\frac{-\sqrt{eg} + \sqrt{-fg - 2g\sqrt{eg} \times z^{\frac{1}{n}} + gz^2}}{\sqrt{g}}$ & $\frac{-\sqrt{-eg} - \sqrt{fg - 2g\sqrt{eg} \times z^{\frac{1}{n}} + gz^2}}{\sqrt{g}}$

& horum pari alterutro curvam metiri licet. Hic primum tantum divisorum par illustrabo, quo res perficienda est, quando quantitas $e + fz^n + gz^{2n}$ dividi non potest in duo binomia. Quoniam hi divisores factores duo sunt quantitatis hujus $e + fz^n + gz^{2n}$, sequitur $\frac{dz^{\frac{1}{n}-1}}{e + fz^n + gz^{2n}}$ esse $=$

$\frac{g dz^{\frac{1}{n}-1}}{\sqrt{eg} + \sqrt{-fg + 2g\sqrt{eg} \times z^{\frac{1}{n}} + gz^2} \times \sqrt{eg} - \sqrt{-fg + 2g\sqrt{eg} \times z^{\frac{1}{n}} + gz^2}}$,

vel (ponendo $\sqrt{eg} = r$, & $\sqrt{-fg + 2g\sqrt{eg}} = s$) $=$

$\frac{g dz^{\frac{1}{n}-1}}{r + sz^{\frac{1}{n}} + gz^2 \times r - sz^{\frac{1}{n}} + gz^2}$. Jam, non ponendum est, sicut in

casu priori, $\frac{1}{r + sz^{\frac{1}{n}} + gz^2 \times r - sz^{\frac{1}{n}} + gz^2} = \frac{m}{r + sz^{\frac{1}{n}} + gz^2}$

$+ \frac{n}{r - sz^{\frac{1}{n}} + gz^2}$, quoniam reducendo partem posteriorem hujus æquationis ad eundem denominatorem cum priori habebimus plures terminos, quam requiruntur ad determinandas coefficientes assumptas m & n; ponendum autem est $\frac{1}{r + sz^{\frac{1}{n}} + gz^2 \times r - sz^{\frac{1}{n}} + gz^2} =$

$\frac{m}{r + sz^{\frac{1}{n}} + gz^2} + \frac{pz^{\frac{1}{n}}}{r + sz^{\frac{1}{n}} + gz^2} + \frac{n}{r - sz^{\frac{1}{n}} + gz^2} + \frac{qz^{\frac{1}{n}}}{r - sz^{\frac{1}{n}} + gz^2}$, &

hic inveniemus $m = n = \frac{1}{2r}$, & $p = -q = \frac{1}{2rs}$: adeo ut

$\frac{dz^{\frac{1}{n}-1}}{e + fz^n + gz^{2n}}$ fit $= \frac{g}{2r} \times \frac{dz^{\frac{1}{n}-1}}{r + sz^{\frac{1}{n}} + gz^2} + \frac{g}{2rs} \times \frac{dz^{n-1}}{r + sz^{\frac{1}{n}} + gz^2} +$

$\frac{g}{2r} \times \frac{dz^{\frac{1}{n}-1}}{r - sz^{\frac{1}{n}} + gz^2} - \frac{g}{2rs} \times \frac{dz^{n-1}}{r - sz^{\frac{1}{n}} + gz^2}$.

Porro si ordinata sit $\frac{dz^{\frac{1}{n}-1}}{e + fz^n + gz^{2n}}$ seu $\frac{dz^{\frac{1}{n}-1}}{e + fz^n + gz^{2n}}$, quando tri-

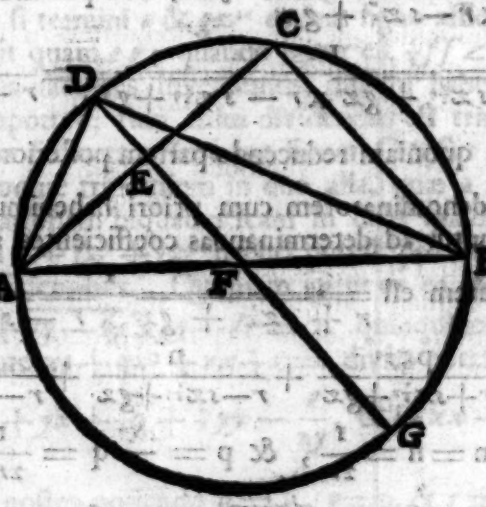
trinominm $e + fz^n + gz^{2n}$ in duo binomia dividi non potest, curva metienda est idem dividendo in quatuor trinomia, quod fit dividendo illud primum in duo trinomia, & deinde singula ista trinomia

in

in dua alia. Et eodem modo metiri licet curvam, cujus ordinata sit $\frac{dz^{\frac{\delta}{\lambda}-1}}{e+fz^{\lambda}+gz^{2\lambda}}$, quamvis $\frac{1}{4}ff$ minor sit quam eg, si δ sit numerus quilibet integer, & λ quicunque numerorum in dupla analogia à binario incipientium *.

HACTENUS *Coteſius* progreſſus eſt. Eſt autem & ratio dividendi trinomium, quando in duo binomia reſolvi non poteſt, in numerum quemlibet trinomiorum, quam nunc tibi exponam. Primum vero id, quod *Coteſius* effecit, via paulo diverſa exequar.

DIAMETRO AB deſcribatur circulus ACB, & in ejus circumferentia ſumpto puncto C, ducantur AC, BC. Deinde diviſo arcu AC in partes duas æquales in puncto D ducantur AD, DB & diameter $DEFG$. Sit radius $AF=1$, $AD=m$, $BD=n$. Eſt rectangu-



lum ſub $GDE\ (=2\times DE)=ADq=mm$; ideoque $EF=\frac{2-mm}{2}$, & $BC=2EF=2-mm$. Item cum ſit $mm=ADq=ABq-BDq=4-nn$, erit quoque $BC=nn-2$.

* Vid. *Coteſii* oper. part. prior. pag. 115.

JAM

JAM vero $\overline{1+mx+xx} \times \overline{1-mx+xx} =$ eſt $1+\overline{2-mm} \times xx + x^4 = 1 + BC \times xx + x^4$, & $\overline{1+nx+xx} \times \overline{1-nx+xx} = 1 + \overline{2-nn} \times xx + x^4 = 1 - BC \times xx + x^4$. Si igitur detur trinomium $e \pm fz^r + gz^{2r}$ in duo alia dividendum, primum per g dividatur, ut reducatur ad hoc $g \times \frac{e}{g} \pm \frac{f}{g} z^r + z^{2r}$; deinde ſi $\frac{1}{4}ff <$ eſt eg, quo fit, ut $\frac{1}{2}f <$ ſit $\sqrt{eg}$, & $\frac{f}{g} < 2\sqrt{\frac{e}{g}}$; ſumatur $\sqrt{\frac{e}{g}}$ pro unitate ſive radio circuli ACB, & $BC = \frac{f}{g}$, deinde erit $\frac{e}{g} + \frac{f}{g} z^r + z^{2r} = \overline{1+AD} \times z^{\frac{1}{2}r} + z^r \times \overline{1-AD} \times z^{\frac{1}{2}r} + z^r$, & $\frac{e}{g} - \frac{f}{g} z^r + z^{2r} = \overline{1+BD} \times z^{\frac{1}{2}r} + z^r \times \overline{1-BD} \times z^{\frac{1}{2}r} + z^r$.

EODEM modo unumquodque horum trinomiorum in duo alia dividere potes, trinomiaque inde inventa in duo alia, & ita procedere ad infinitum; eaque ratione omne, quod *Coteſius* noſter præſtitit, arcus circuli diviſione efficere licet.

PROXIMUM vero docebo, quomodo arcus circuli diviſione trinomium in numerum quemlibet imparem trinomiorum dividatur.

PROPONATUR autem primum trinomium in tria alia reſolvere. In hunc finem quantitati $1 + bx + cx^2 + dx^3 + cx^4 + bx^5 + x^6$ diviſorem trinomium ratione ſequenti inveniemus. Sumatur trinomium $1 - mx + xx$, & hoc trinomio dividatur quantitas propoſita, donec tantum non reſtat invenienda dimidia pars terminorum, qui quotientem conſtituerent, ſi diviſio ad finem perduceretur, hoc modo,

$1-mx+xx) \; 1+bx+cx^2+dx^3+cx^4+bx^5+x^6 \; (1+\overline{b+m}\times x+\overline{c-1+bm+m^2}\times x^2$

$1-mx+xx$

$\overline{b+m}\times x+\overline{c-1}\times x^2+dx^3+cx^4+bx^5+x^6$

$\overline{b+m}\times x-\overline{bm-mm}\times x^2+\overline{b+m}\times x^3$

$\overline{c-1+bm+mm}\times x^2+\overline{d-b-m}\times x^3+cx^4+bx^5+x^6$

$\overline{c-1+bm+mm}\times x^2-\overline{cm+m-bm^2-m^3}\times x^3+\overline{c-1+bm+m^2}\times x^4$

$\overline{d-b-2m+cm+bm^2+m^3}\times x^3+1-bm-mm\times x^4+bx^5+x^6$

pone $d - b - 2m + cm + bm^2 + m^3$ coefficientem termini primi eorum, quibus conſtat reſiduum poſt diviſionem reſtans, æqualem $b + m$ coefficienti termini præcedentis ultimo in quotiente, & diviſio ad finem

nem hac ratione perduci poteſt

$$1-mx+xx\,)\,\overline{d-b-2m+cm+bmm+m^3\times x^3+1-bm-mm\times x^4+bx^5+x^6}\,(b-m\times x^3+x^4$$
$$\overline{b+m\times x^3} \qquad\qquad \overline{-bm-m^2\times x^4+b+m\times x^5}$$
$$x^4-mx^5+x^6$$
$$x^4-mx^5+x^6$$
$$0\quad 0\quad 0$$

ſed cum oportet, ut ſit $d-b-2m+cm+bmm+m^3=b+m$, hanc æquationem habebimus $d-2b=3-c\times m-bm^2-m^3$: unde m coefficiens termini ſecundi diviſoris $1-mx+xx$ invenietur *.

JAM ſi b & c uterque ponatur $=0$, quantitas dividenda fiet trinomia $1+dx^3+x^6$, & m coefficiens termini ſecundi trinomii $1-mx+xx$, quod hoc alterum dividit, determinabitur hac æquatione $d=3m-m^3$, vel $m^3-3m+d=0$. In hac autem æquatione, quando $d<$ eſt 2, ſi in circulo, qui radio unitati æquali deſcribatur, chorda ducatur æqualis d, invenientur radices m æquationis $m^3-3m+d=0$ dividendo in tres partes æquales arcus, quibus ſubtenditur chorda d. Hinc tres diviſores trinomii $1+dx^3+x^6$, exiſtente $d<2$, ſic invenientur. Radio unitati æquali deſcribatur circulus $ABCD$, & in eo applicetur chorda $AC=d$. Dividantur arcus ABC, ADC, quibus hæc chorda ſubtenditur, in tres partes æquales in punctis E, F, G, H, chordis AE, AH ductis, quibus factis unumquodque horum trinomiorum $1-AE\times x+xx$, $1-AH\times x+xx$, & $1+AE+AH\times x+xx$ quantitatem $1+dx^3+x^6$ dividet; propterea quod AE, AH radices ſunt affirmativæ, & $AE+AH$ radix negativa æquationis $m^3-3m+d=0$.

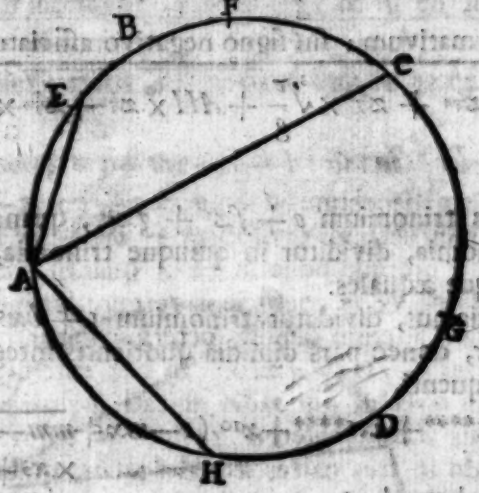
PORRO hæc trinomia in ſe ducendo, quantitatem $1+dx^3+x^6$ produces. Ponantur enim $AE=m$, $AH=n$, $AE+AH=p$, in ſe ducendo hæc trinomia quantitatem invenies

$$\begin{array}{lllll} 1-mx+3xx & -2mx^3+3x^4 & -mx^5+x^6 \\ -n & +mn & -2n & -mp-n \\ +p & -mp & +2p & -np+p \\ & -np & +mnp & +mn \end{array}$$

quoniam igitur m, n, $-p$ radices ſunt æquationis $m^3-3m+d=0$, & in omni æquatione termini ſecundi coefficiens ſigno mutato ſummam radicum adæquat, termini tertii coefficiens ſummæ æqualis eſt

* Vid. de Moire *of Chances*, pag. 113.

productorum, quæ oriuntur ducendo in ſe binas radices, quæ ſingulam binarium radicum combinationem conſtituunt, & termini quarti coefficiens ſigno mutato omnibus productis, quæ oriuntur ducendo in ſe ternas



radices ſingulam ternariam radicum combinationem efficientes; hic inveniemus $m+n-p=0$, $mn-mp-np=-3$, & $-mnp=-d$; ideoque quantitas, quam produximus ducendo in ſe trinomia, quorum termini medii coefficientes habuerunt $-m$, $-n$, p, invenietur æqualis $1+dx^3+x^6$.

Si quantitas fuiſſet $1-dx^3+x^6$, eam dividendo per $1+mx+xx$ produceretur eadem æquatio ut prius, ideoque $1+AE\times x+xx$, $1+AH\times x+xx$, & $1-\overline{|AE+AH}\times x+xx$ erunt tres trinomii factores quantitatis $1-dx^3+x^6$.

Hinc apparet, quomodo reſolvatur quantitas $e+fz^3+gz^{12}$ in tres factores trinomios, nam $e+fz^3+gz^{12}$ exiſtente $=g\times\overline{\frac{e}{g}+\frac{f}{g}z^3+z^{12}}$, ſi radius circuli ſumatur $=\sqrt[6]{\frac{e}{g}}$ & chorda $AC=\frac{f}{g\sqrt[3]{\frac{e}{g}}}=\frac{f}{\sqrt[3]{egg}}$, quando $\frac{f}{\sqrt[3]{egg}}$ eſt $<2\sqrt[6]{\frac{e}{g}}$, hoc eſt $f<2\sqrt{e^3g^3}$ vel $2\sqrt{eg}$, & proinde $\frac{1}{4}ff<eg$, quo trinomium dividi non poteſt in duo binomia,

mia, habebitur $e + fz^{n} + gz^{2n} = g \times \sqrt[3]{\frac{e}{g} - AE} \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[3]{\frac{e}{g}} - AH \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[3]{\frac{e}{g}} + \sqrt{AE + AH} \times z^{\frac{1}{n}} + z^{\frac{2}{n}}$, si f habet signum affirmativum; sin signo negativo afficiatur, idem erit $= g \times \sqrt[3]{\frac{e}{g}} + AE \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[3]{\frac{e}{g}} + AH \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[3]{\frac{e}{g}} - \sqrt{AE + AH} \times z^{\frac{1}{n}} + z^{\frac{2}{n}}$.

SIMILITER trinomium $e + fz^{n} + gz^{2n}$, quando dividi non potest in duo binomia, dividitur in quinque trinomia, dividendo arcum in partes quinque æquales.

UT hoc faciamus, dividatur trinomium $1 + bx^{5} + x^{10}$ hoc altero $1 + mx + xx$, donec pars dimidia quotientis integræ proxime superatur, modo sequenti.

$1+mx+xx) \; 1^{****} + bx^{5****} + x^{10} \; (1 - mx + \sqrt{mm-1} \times x^{2} - \sqrt{m^{3}-2m} \times x^{3} + m^{4} - 3m^{2} + 1 \times x^{4}$

$1 + mx + x^{2}$

$-mx - x^{2}$

$-mx - mmx^{2} - mx^{3}$

$-1 + mm \times x^{2} + mx^{3}$

$-1 + mm \times x^{2} + m^{3} - m \times x^{3} + mm - 1 \times x^{4}$

$-m^{3} + 2m \times x^{3} - mm + 1 \times x^{4} + bx^{5}$

$-m^{3} + 2m \times x^{3} - m^{4} + 2m^{2} \times x^{4} - m^{3} + 2m \times x^{5}$

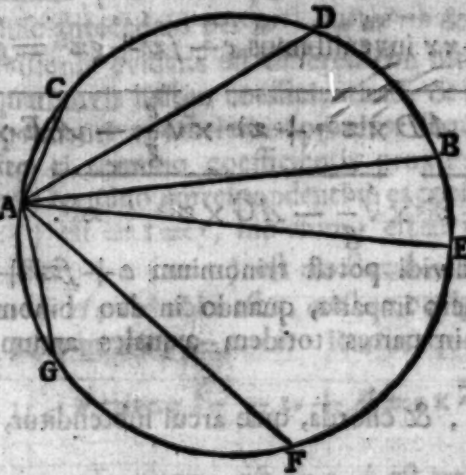
$m^{4} - 3m^{2} + 1 \times x^{4} + m^{3} - 2m + b \times x^{5}$

$m^{4} - 3m^{2} + 1 \times x^{4} + m^{5} - 3m^{3} + m \times x^{5} + \&c$

$-m^{5} + 4m^{3} - 3m + b \times x^{5} - \&c$

ponendo coefficientem termini primi hujus residui æqualem coefficienti termini penultimi quotientis, hanc habebimus æquationem $b = 5m - 5m^{3} + m^{5}$; in qua æquatione, b existente < 2, radix m invenietur dividendo in partes quinque æquales arcum, cujus chorda est b, semidiametro circuli existente unitate. Hinc trinomium $e + fz^{n} + gz^{2n}$, quando in duo binomia dividi non potest, in quinque trinomia dividitur; dividendo enim hoc trinomium per g & æquando quotientem trinomio $1 + bx^{5} + x^{10}$, invenietur $z^{\frac{1}{n}} = x$, $\sqrt[10]{\frac{e}{g}} = 1$, & $\frac{f}{\sqrt[5]{eeg^{3}}} = b$. Sed existente

istente $\frac{1}{4}f < \sqrt{eg}$ $(= \sqrt{eegg})$, $\frac{\frac{1}{7}f}{\sqrt{e^2g^3}}$ $(= \frac{\frac{1}{7}f}{\sqrt{e^4g^6}})$ est $< \sqrt[10]{\frac{e}{g}}$. Si circulus igitur describatur intervallo $AB = \sqrt[10]{\frac{e}{g}}$, & in eo sumatur chorda $AB = \frac{f}{\sqrt{eegg}}$, deinde arcus $AC =$ parti quintæ arcus AB, & dividatur circulus in quinque partes æquales in punctis C, D, E, F, G, si denique chordæ ducantur AC, AD, AE, AF, AG; quoniam omnes hæ lineæ AC, AD, AE, AF, AG radices sunt æquationis



$b = 5m - 5m^3 + m^5$, trinomium $\frac{e}{g} + \frac{f}{g}z^{10} + z^{20}$ unoquoque horum dividetur $\sqrt[5]{\frac{e}{g}} + AC \times z^{10} + z^{20}$, $\sqrt[5]{\frac{e}{g}} - AD \times z^{10} + z^{20}$, $\sqrt[5]{\frac{e}{g}} + AE \times z^{10} + z^{20}$, $\sqrt[5]{\frac{e}{g}} - AF \times z^{10} + z^{20}$, $\sqrt[5]{\frac{e}{g}} + AG \times z^{10} + z^{20}$.

Ut determinetur signum termini secundi uniuscujusque horum factorum, observandum est singulas chordas AC, AD, AE, AF, AG radices esse æquationis $b = 5m - 5m^3 + m^5$, unde radices affirmativæ a negativis distingui possunt ope propositionis eximiæ magni geome-

træ

træ *Jacobi Gregorii* *, qui oſtendit $AC + AG + AE$ eſſe $= AF + AD$. Cum igitur æquationi ſecundus terminus deſit, quo fit, ut aggregatum radicum affirmativarum & negativarum ſit æquale, quoniam AC eſt radix affirmativa, erunt quoque AG & AE radices affirmativæ, & AD, AF radices negativæ.

JAM hi quinque diviſores quuntitatis $\frac{e}{g} + \frac{f}{g}z^n + z^{2n}$ in ſe ducti iſtam quantitatem producunt; quod comprobare potes conferendo coefficientes quantitatis inde productæ cum æquatione $m^5 - 5m^3 + 5m - b = 0$, ut antea factum fuit, ubi trinomium in tria alia diviſimus.

SI ordinata fuiſſet $\frac{dz^{\frac{1}{n}-1}}{e - fz^n + gz^{2n}}$, dividendo trinomium $1 - bx^5 + x^{10}$ per $1 - mx + xx$ inveniſſemus $e - fz^n + gz^{2n} = g \times \sqrt[5]{\frac{e}{g}} - AC \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[5]{\frac{e}{g}} + AD \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[5]{\frac{e}{g}} - AE \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[5]{\frac{e}{g}} + AF \times z^{\frac{1}{n}} + z^{\frac{2}{n}} \times \sqrt[5]{\frac{e}{g}} - AG \times z^{\frac{1}{n}} + z^{\frac{2}{n}}$.

GENERATIM dividi poteſt trinomium $e + fz^n + gz^{2n}$ in quotlibet trinomia, numero imparia, quando in duo binomia reſolvi non poteſt, dividendo in partes totidem æquales arcum circuli, cujus ſemidiameter eſt $\frac{e}{g}\Big|^{\frac{1}{2\lambda}}$, & chorda, quæ arcui ſubtenditur, $= \frac{f}{g \times \frac{e}{g}\Big|^{\frac{\lambda-1}{2\lambda}}}$ $= \frac{f}{e^{\frac{\lambda-1}{2\lambda}} \times g^{\frac{\lambda+1}{2\lambda}}}$; litera λ numerum denotante factorum, in quos trinomium propoſitum reſolvendum eſt.

HOC ſatis comprobabitur oſtendendo trinomium $1 \pm bx^{\lambda} + x^{2\lambda}$ reſolvi poſſe, quando λ eſt numerus impar & $b < 2$, in tot trinomia, quot ſunt unitates in λ, arcus diviſione; quod ratione ſequenti fiet. Trinomii propoſiti diviſio a trinomio $1 - mx + xx$ inchoetur, & coefficientes terminorum quotientis in ordinem ſequentem redigantur.

* Geometr. part. univerſal. prop. 69.

$$\begin{array}{l}
1\\
m\\
m^2-1\\
m^3-2m\\
m^4-3m^2+1\\
m^5-4m^3+3m\\
m^6-5m^4+6m^2-1\\
m^7-6m^5+10m^3-4m\\
m^8-7m^6+15m^4-10m^2+1
\end{array}$$

Jam ex methodo *Taylori* denotetur quælibet harum coefficientium promiscue per $m^{\nu}-am^{\nu-2}+bm^{\nu-4}-cm^{\nu-6}+$ &c coefficiens proxime sequens per $m^{\nu'}-a'm^{\nu'-2}+b'm^{\nu'-4}+c'm^{\nu'-6}+$ &c coefficiens proxime præcedens per $m^{\nu}-a'm^{\nu-2}+b'm^{\nu-4}-$ &c & coefficiens huic antecedens per $m^{\nu''}-a''m^{\nu''-2}$ &c; & ita de cæteris. Jam natura divisionis evidens est differentiam inter numerum appositum termino quarumvis harum coefficientium, & numerum appositum termino correspondenti coefficientis proxime sequentis esse æqualem numero appositi ei termino coefficientis proxime præcedentis, qui proxime antecedit termino correspondenti in ea coefficiente præcedente. Sic $a'-a$ $(=a)$ est $=1=\nu$, hic enim ν est $=1$, $b'-b$ $(=b)$ est $=a'=a-1$ & $c'-c$ $(=c)$ est $=b'=b-a''$; sed a existente $=\nu$, a est $=\nu+A$; sed ubi ν est 1, a est $=0$; igitur $1+A$ est $=0$, & $A=-1$; adeo ut a est $=\nu-1$. Rursus quoniam b est $=a-1$ $=\nu-2=\nu-2\nu$, b erit $=\frac{1}{2}\nu\nu-2\nu+A=\nu\times\frac{\nu-1}{2}-2\nu+A=\nu\times\frac{\nu-5}{2}+A$: quando autem ν est $=3$, b est $=0$; ergo A est $=3$, & $b=\nu\times\frac{\nu-5}{2}+3=\overline{\nu-2}\times\frac{\nu-3}{2}$. Porro cum c est $=b-a''=\frac{1}{2}\nu\nu-2\nu+3-a''=\frac{1}{2}\nu\nu-2\nu+3-\nu+3=\frac{1}{2}\nu\nu-3\nu+6=\frac{1}{2}\nu\nu-3\nu+6\nu$, c erit $=\frac{1}{2\times3}\nu\nu\nu-\frac{1}{2}\nu\nu+6\nu+A=\nu\times\frac{\nu-1}{2}\times\frac{\nu-2}{3}-3\nu\times\frac{\nu-1}{2}+6\nu+A$. Ubi autem ν est $=5$, c est $=0$; ergo A est $=-10$, & $c=\nu\times\frac{\nu-1}{2}\times\frac{\nu-2}{3}-3\nu\times\frac{\nu-1}{2}+6\nu-10=\overline{\nu-3}\times\frac{\nu-4}{2}$

$\frac{\nu-4}{2} \times \frac{\nu-5}{3}$, & ita de cæteris; adeo ut hæ coefficientes generatim exprimentur ſequenti ſerie $m^\nu - \sqrt{\nu-1} \times m^{\nu-2} + \overline{\nu-2} \times \frac{\nu-3}{2} \times m^{\nu-4} - \sqrt{\nu-3} \times \frac{\nu-4}{2} \times \frac{\nu-5}{3} \times m^{\nu-6} +$ &c.

PORRO ſi diviſio continuetur, donec tantum non reſtat dimidia pars quotientis integræ, index ν termini ultimo inventi in quotiente erit $= \lambda - 1$, & primus reſidui terminus erit æqualis termino, qui proxime oriretur in quotiente, ſi terminus $\pm b x^\lambda$ non fuiſſet in quantitate dividenda, una cum $\pm b$ coefficiente illius termini, ſed in proxime ſequenti quotientis termino index ν erit $= \lambda$. Hic autem primus reſidui terminus poni debet æqualis termino quotientis proxime præcedenti illo termino, quo primum ſuperatur totius quotientis pars dimidia, in quo termino index ν eſt $= \lambda - 2$, hinc $\pm b + m^\lambda - \sqrt{\lambda-1} \times m^{\lambda-2} + \overline{\lambda-2} \times \frac{\lambda-3}{2} m^{\lambda-4} - \sqrt{\lambda-3} \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} m^{\lambda-6} +$ &c eſt $= m^{\lambda-2} - \sqrt{\lambda-3}\, m^{\lambda-4} + \overline{\lambda-4} \times \frac{\lambda-5}{2} m^{\lambda-6} -$ &c igitur b eſt $= \mp m^\lambda \pm m^{\lambda-2} \mp \lambda \times \frac{\lambda-3}{2} m^{\lambda-4} \pm \lambda \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} m^{\lambda-6} \pm$ &c.

HINC b exiſtente < 2, manifeſtum eſt, corollario primo ſecundæ noſtræ propoſitionis, m aut affirmative aut negative ſumptam eſſe chordam arcus, cujus multiplex pro numero unitatum in λ, habet b pro ſua chorda, hæc chorda erit radix affirmativa æquationis, ſi terminus ultimus ad dextram ſit affirmativus, ſin ex contrario, ſi is terminus ſit negativus, erit chorda radix æquationis negativa. Dividendo autem circulum totum in tot partes æquales quot ſunt unitates in λ, ut antea fecimus *, omnes æquationis hujus radices inveniemus, quibus conficere licet tot trinomia, quot ſunt unitates in λ, quorum unumquodque trinomium $1 \pm b x^\lambda + x^{2\lambda}$ dividet; & omnia in ſe ducta iſtud trinomium efficient.

HOC autem poſtremum ratione ſequenti manifeſtum fiet. Sint m, n, p, q, r, &c radices æquationis $m^\lambda - \lambda m^{\lambda-2} + \lambda \times \frac{\lambda-3}{2} m^{\lambda-4} - \lambda \times \frac{\lambda-4}{2} \times \frac{\lambda-5}{3} m^{\lambda-6} +$ &c .... $\mp b = 0$. Deinde ſi fiant $\lambda - 1 = \mu$,

* Pag. 43.

$\lambda - 2$

$\lambda-2=\iota$, $\lambda-3=\pi$, $\lambda-4=\rho$, &c; item ſumma omnium quantitatum m, n, p, q, r, &c $=\varsigma$, ſumma omnium rectangulorum ſub ſingulis binis $=d$, ſumma omnium productorum a ſingulis ternis $=e$, ſumma omnium productorum a ſingulis quaternis $=f$, ſumma omnium productorum a ſingulis quinis $=g$, &c; erit $1+mx+xx\times 1+nx+xx\times 1+px+xx\times 1+qx+xx\times 1+rx+xx\times$ &c $=$

$$1+\lambda x^{2}+\lambda x\frac{\lambda-1}{2}x^{4}+\lambda x\frac{\lambda-1}{2}\times\frac{\lambda-2}{3}x^{6}+\lambda x\frac{\lambda-1}{2}\times\frac{\lambda-2}{3}\times\frac{\lambda-3}{4}x^{8}+\left[\frac{\lambda-1}{2}\times\frac{\lambda-2}{3}\times\frac{\lambda-3}{4}\times\frac{\lambda-4}{5}x^{10}+\text{&c}\right.$$

$$+\varsigma x+\mu\varsigma x^{3}+\mu x\frac{\mu-1}{2}\varsigma x^{5}+\mu x\frac{\mu-1}{2}\times\frac{\mu-2}{3}\varsigma x^{7}+\mu x\frac{\mu-1}{2}\times\frac{\mu-2}{3}\times\frac{\mu-3}{4}\varsigma x^{9}+\text{&c}$$

$$=+dx^{2}+\nu dx^{4}+\nu x\frac{\nu-1}{2}dx^{6}+\nu x\frac{\nu-1}{2}\times\frac{\nu-2}{3}dx^{8}+\text{&c}=$$

$$+ex^{3}+\pi ex^{5}+\pi x\frac{\pi-1}{2}ex^{7}+\text{&c}$$

$$+fx^{4}+\rho fx^{6}+\text{&c}$$

$$+gx^{5}+\text{&c}$$

$$=+\text{&c}$$

Jam conferendo terminos ς, d, e, f, g, &c cum coefficientibus terminorum in æquatione præcedente, inveniemus $\varsigma=0$, $d=-\lambda$, $e=0$, $f=\lambda\times\frac{\lambda-3}{2}$, &c. Hinc autem inveniemus $\varsigma x=0$; $\mu\varsigma+e\times xx=0$; $\lambda+d\times x^{3}=\lambda-\lambda\times x^{3}=0$; $\lambda\times\frac{\lambda-1}{2}+\nu d+f\times x^{4}=\lambda\times\frac{\lambda-1}{2}-\lambda\times\overline{\lambda-2}+\lambda\times\frac{\lambda-3}{2}\times x^{4}=0$; & ita in cæteris, donec ad medium terminum pervenitur quantitatis oriundæ trinomia ſupra poſita in ſe ducendo. Hunc autem medium terminum inveniemus $=\pm bx^{\lambda}$. Cæteri autem omnes hujus quantitatis termini poſt medium, præter ultimum, qui eſt $=x^{2\lambda}$, erunt $=0$; propterea quod omnium terminorum æqualiter a medio diſtantium coefficientes æquales ſunt. Hinc autem manifeſtum eſt quantitatem hanc trinomio $1\pm bx^{\lambda}+x^{2\lambda}$ eſſe æqualem.

Hac ratione dividitur omne trinomium in numerum quemcunque imparem trinomiorum. Si requiritur in numerum parem trinomium dividere, dividendum eſt in duo, & iſta in duo alia, donec tandem perveniatur ad trinomia in numerum trinomiorum imparem dividenda. Sunt quidem & alii modi, licet minus ſint generales, trinomium in alia trinomia numero paria dividendi; huc enim ſpectat *corollar.* 2. *propoſit.*

*posit. 2. præced.** Et in hoc negotio utilia quoque erunt alia quæ de arcuum circuli divisione geometræ scripserunt. Hæc autem mitto, exponere properans eorum, quæ jam tradidi, usum.

HINC vero aperitur via omnem curvam metiendi hujus formæ $\frac{dx^{\frac{s}{\lambda}n-1}}{e+fx^n+gx^{2n}}$, quando denominator in duo binomia dividi non potest, denominatorem in tot trinomia dividendo, quot sunt unitates in λ; nam postquam hoc fit, ordinata ad simpliciores facile reducitur. In quem finem sequentia inveni, si ponatur $\frac{1}{1+qx+xx \times 1+rx+xx} = \frac{K+kx}{1+qx+xx} + \frac{L+lx}{1+rx+xx}$, erit $K = \frac{q}{q-r}$, $k = \frac{1}{q-r}$, $L = \frac{r}{r-q}$, & $l = \frac{1}{r-q}$; si ponatur $\frac{1}{1+qx+xx \times 1+rx+xx \times 1+sx+xx} = \frac{K+kx}{1+qx+xx} + \frac{L+lx}{1+rx+xx} + \frac{M+mx}{1+sx+xx}$, erit $K = \frac{qq-1}{q-r \times q-s}$, $k = \frac{q}{q-r \times q-s}$, $L = \frac{rr-1}{r-q \times r-s}$, $l = \frac{r}{r-q \times r-s}$, $M = \frac{ss-1}{s-q \times s-r}$, $m = \frac{s}{s-q \times s-r}$; item si ponatur $\frac{1}{1+qx+xx \times 1+rx+xx \times 1+sx+xx \times 1+tx+xx} = \frac{K+kx}{1+qx+xx} + \frac{L+lx}{1+rx+xx} + \frac{M+mx}{1+sx+xx} + \frac{N+nx}{1+tx+xx}$, erit $K = \frac{q^3-2q}{q-r \times q-s \times q-t}$, $k = \frac{qq-1}{q-r \times q-s \times q-t}$, $L = \frac{r^3-2r}{r-q \times r-s \times r-t}$, $l = \frac{rr-1}{r-q \times r-s \times r-t}$, $M = \frac{s^3-2s}{s-q \times s-r \times s-t}$, $m = \frac{ss-1}{s-q \times s-r \times s-t}$, $N = \frac{t^3-2t}{t-q \times t-r \times t-s}$, $n = \frac{tt-1}{t-q \times t-r \times t-s}$; si vero ponatur

* Pag. 17.

$$\frac{1}{1+qx+xx\times 1+rx+xx\times 1+sx+xx\times 1+tx+xx\times 1+vx+xx} = \frac{K+kx}{1+qx+xx} + \frac{L+lx}{1+rx+xx} + \frac{M+mx}{1+sx+xx} + \frac{N+nx}{1+tx+xx} + \frac{P+px}{1+vx+xx}$$

, erit $K = \frac{q^4-3qq+1}{q-r\times q-s\times q-t\times q-v}$, $k = \frac{q^3-2q}{q-r\times q-s\times q-t\times q-v}$, $L = \frac{r^4-3rr+1}{r-q\times r-s\times r-t\times r-v}$, $l = \frac{r^3-2r}{r-q\times r-s\times r-t\times r-v}$, $M = \frac{s^4-3ss+1}{s-q\times s-r\times s-t\times s-v}$, $m = \frac{s^3-2s}{s-q\times s-r\times s-t\times s-v}$, $N = \frac{t^4-3tt+1}{t-q\times t-r\times t-s\times t-v}$, $n = \frac{t^3-2t}{t-q\times t-r\times t-s\times t-v}$, $P = \frac{v^4-3vv+1}{v-q\times v-r\times v-s\times v-t}$, $p = \frac{v^3-2v}{v-q\times v-r\times v-s\times v-t}$; & eodem modo in aliis. Lex autem, quam hæ quantitates obſervant, manifeſta eſt; in denominatoribus res nulla explicatione eget; numeratores autem eadem ratione formantur, ac coefficientes terminorum, qui oriuntur dividendo unitatem per trinomium *; ideoque ex iis, quæ ſupra oſtendimus †, generatim determinantur.

Ita caſus curvarum, quarum ordinatæ habent denominatores trinomios, generatim perficiuntur. A trinomiis ordinatis ad quadrinomias progreditur *Coteſius*. Quid autem ſuper his præſtitit, verbo tibi expediam, cum earum tractandi ratio poſt ea, quæ de trinomia ordinata ſcripſi, ſponte ſe offerat; facile enim reſolvitur quadrinomius denominator in factores binomios & trinomios; ponendo eum $=0$; nam ſi æquatio hinc formata tres habet radices, hæ radices præbent tot diviſores binomios; ſin unicam tantum radicem habet, qua exprimi poteſt valor z, ea radix exhibebit unum factorem, & diviſio quadrinomii per hunc factorem præbebit trinomium pro altero ejus factore. Et hæc æquatio unam ſemper habebit radicem, quoniam cubicæ formam induit.

* Vid. Pag. 45. † Pag. 46.

Si curvæ ordinata sit $\frac{dz^{b-1}}{e+fz^{a}+gz^{2a}+hz^{3a}} = \frac{\frac{d}{b}z^{b-1}}{\frac{e}{b}+\frac{f}{b}z^{a}+\frac{g}{b}z^{2a}+z^{3a}}$,

& æquatio $\frac{e}{b}+\frac{f}{b}z^{a}+\frac{g}{b}z^{2a}+z^{3a}=0$ tres habeat radices, ordinata ad alias cum denominatoribus binomiis facile reducitur: si enim generatim ponatur $\frac{1}{p+z^{a}\times q+z^{a}\times r+z^{a}\times s+z^{a}\times \&c} = \frac{k}{p+z^{a}} + \frac{l}{q+z^{a}} + \frac{m}{r+z^{a}} + \frac{n}{s+z^{a}} + \&c$, supputandi ratione supra * expositа invenies, & olim ostensum fuit †, $k=\frac{1}{q-p\times r-p\times s-p\times \&c}$, $l=\frac{1}{p-q\times r-q\times s-q\times \&c}$, $m=\frac{1}{p-r\times q-r\times s-r\times \&c}$, $n=\frac{1}{p-s\times q-s\times r-s\times \&c}$, &c. Unde etiam, si ponatur $\frac{1}{P+pz^{a}\times Q+qz^{a}\times R+rz^{a}\times S+sz^{a}\times \&c} = \frac{k}{P+pz^{a}} + \frac{l}{Q+qz^{a}} + \frac{m}{R+rz^{a}} + \frac{n}{S+sz^{a}} + \&c$, & λ sit binomiorum numerus, invenies $k=\frac{p^{\lambda-1}}{Qp-Pq\times Rp-Pr\times Sp-Ps\times \&c}$, $l=\frac{q^{\lambda-1}}{Pq-Qp\times Rq-Qr\times Sq-Qs\times \&c}$, $m=\frac{r^{\lambda-1}}{Pr-Rp\times Qr-Rq\times Sr-Rs\times \&c}$, $n=\frac{s^{\lambda-1}}{Ps-Sp\times Qs-Sq\times Rs-Sr\times \&c}$, &c.

Si æquatio $\frac{e}{b}+\frac{f}{b}z^{a}+\frac{g}{b}z^{2a}+z^{3a}=0$ non nisi unam habeat radicem, puta $\frac{k}{l}$, dividendo ordinatæ denominatorem $e+fz^{a}+gz^{2a}+hz^{3a}$

* Pag. 34, 35.

† Vid. Act. Erudit. Ann. 1702. pag. 212.

per

per $z^{\nu} - \frac{k}{l}$ in binomium & trinomium factorem resolves. Si quidem divisionem a quantitate e inchoes, trinomium invenies $-e\frac{l}{k} - \left|f\frac{l}{k} + e\frac{ll}{kk} \times z^{\nu} + hz^{2\nu}\right.$; incipiendo autem ab altera quadrinomii parte trinomium prodibit $hz^{2\nu} + g + b\frac{k}{l} \times z^{\nu} + f + g\frac{k}{l} + b\frac{kk}{ll}$. Jam si ponamus, ut fit in *Cotesii* opere, $p + qz^{\nu} + hz^{2\nu}$ pro trinomio hoc factore, erit $p = -e\frac{l}{k}$ vel $f + g\frac{k}{l} + b\frac{kk}{ll}$, & $q = -e \times \frac{ll}{kk} - f\frac{l}{k}$ vel $g + b\frac{k}{l}$; ordinata autem $\frac{dz^{\lambda\nu-1}}{e + fz^{\nu} + gz^{2\nu} + hz^{3\nu}}$ erit $= \frac{dlz^{\lambda\nu-1}}{-k + lz^{\nu} \times p + qz^{\nu} + hz^{2\nu}}$: Et area huic posteriori ordinatæ pertinens determinatur, vel ratione, quam antea * explicuimus, vel ponendo fractionem $\frac{1}{-k + lz^{\nu} \times p + qz^{\nu} + hz^{2\nu}} = \frac{r}{-k + lz^{\nu}} + \frac{s + tz^{\nu}}{p + qz^{\nu} + hz^{2\nu}}$, unde invenietur $r = \frac{ll}{pll + qkl + hkk}$, $s = \frac{-ql - hk}{pll + qkl + hkk}$ & $t = \frac{-hl}{pll + qkl + hkk}$. Prior ratio adhiberi potest, quando θ est numerus integer, sed posterior sequenda est, quando θ numerus est fractus.

Porro, cum doctissimus amicus tuus, cui *Cotesii* opus debemus, mentionem faciat supputandi areas curvarum, quarum ordinatæ denominatores habent ex terminis quinque vel sex compositos, ratio id exequendi breviter indicanda est.

In quinquinomio tres sunt casus; si enim quinquinomium ponatur $= 0$, æquatio inde formata radices quatuor vel duas vel nullas habere potest. In casu primo resolvi potest quinquinomium in quatuor factores binomios. In casu secundo dividitur in duos binomios factores & in trinomium. Verbi causa, sit quinquinomium propositum $e + fz^{\nu} + gz^{2\nu} + hz^{3\nu} + kz^{4\nu}$; si hoc fiat $= 0$, vel, quod idem

* Pag. 7, 8.

est, si ponatur $\frac{e}{k}+\frac{f}{k}z^{\prime}+\frac{g}{k}z^{\prime\prime}+\frac{h}{k}z^{\prime\prime\prime}+z^{\prime\prime\prime\prime}=0$, & hæc æquatio duas habet radices p & q, quinquinomii duo factores binomii erunt $-p+z^{\prime}$, & $-q+z^{\prime}$, trinomius autem factor $\frac{e}{pq}+\frac{h+pk+qk}{pq}\times z^{\prime}+kz^{\prime\prime}$. Denique in casu tertio quinquinomium in duo trinomia dividitur eodem modo, quo ostendi *Cotesium* dividere in duo alia trinomium $e+fz^{\prime}+gz^{\prime\prime}$. Nam ponendo $z^{\prime}=x-\frac{h}{4k}$ quinquinomium propositum in aliud reducemus, cui unus terminus deficit, quod ope æquationis $y^{6}+2qy^{4}+\overline{qq-4s}\times y^{2}-rr=0$ dividi potest in duos trinomios factores; in quibus factoribus substituendo loco x ejus valorem $z^{\prime}+\frac{h}{4k}$ invenientur duo trinomii factores quinquinomii propositi. Et hoc semper fieri potest, nam terminus ultimus æquationis $y^{6}+2qy^{4}+\overline{qq-4s}\times y^{2}-rr=0$ est semper negativus; ideoque æquatio pro cubica habita semper habet radicem affirmativam, proinde y est semper quantitas possibilis.

POSTQUAM vero ordinatæ denominator in binomios & trinomios factores dividitur, ordinata ipsa in alias, denominatores habentes binomios & trinomios, haud difficulter reducitur. Si quatuor sunt factores binomii, res perficitur per modum illum generalem, de quo sermonem jam nunc de quadrinomio agens habui *. Est autem & modus generalis idem peragendi, quando inter factores trinomia occurrunt; qui hac ratione inveniri potest. Quoniam, si sit

$$\frac{1}{p+z^{\prime}\times q+z^{\prime}\times r+z^{\prime}\times s+z^{\prime}\times t+z^{\prime}}=\frac{k}{p+z^{\prime}}+\frac{l}{q+z^{\prime}}+\frac{m}{r+z^{\prime}}+\frac{n}{s+z^{\prime}}+\frac{o}{t+z^{\prime}},$$

quantitates k, l, m, n, o notæ sunt; ponatur $\overline{p+z^{\prime}\times q+z^{\prime}}\ (=pq+\overline{p+q}\times z^{\prime}+z^{\prime\prime})=v+wz^{\prime}+z^{\prime\prime}$, ut sit $v=pq$ & $w=p+q$; item ponatur $\frac{k}{p+z^{\prime}}+\frac{l}{q+z^{\prime}}\ (=\frac{kq+lp+\overline{k+l}\times z^{\prime}}{\overline{p+z^{\prime}\times q+z^{\prime}}})=\frac{a+bz^{\prime}}{v+wz^{\prime}+z^{\prime\prime}}$, ut sit $a=kq+lp$, &

* Pag. 50.

$b=k+l$.

$b=k+l$. Deinde vero erit $\frac{1}{v+wz^2+z^{22}\times r+z^2\times s+z^2\times t+z^2}$
$=\frac{a+bz^2}{v+wz^2+z^{22}}+\frac{m}{r+z^2}+\frac{n}{s+z^2}+\frac{o}{t+z^2}$, ubi, datis v, w, r, s, t, inveniendæ sunt a, b, m, n, o. Primum quæramus a & b; in quem finem observandum est, cum sit $k=\frac{1}{q-p\times r-p\times s-p\times t-p}$ & $l=\frac{1}{p-q\times r-q\times s-q\times t-q}=\frac{-1}{q-p\times s-q\times s-q\times t-q}$, erit $kq+lp=\frac{q\times r-q\times s-q\times t-q-p\times r-p\times s-p\times t-p}{q-p\times r-p\times s-p\times t-p\times r-q\times s-q\times t-q}$
$=\frac{rst\times q-p-rs-rt-st\times q^2-p^2+r+s+t\times q^3-p^3-q^4-p^4}{q-p\times r-p\times s-p\times t-p\times r-q\times s-q\times t-q}$; & erit $k+l=$
$\frac{-rs-rt-st\times q-p+r+s+t\times q^2-p^2-q^3-p^3}{q-p\times r-p\times s-p\times t-p\times r-q\times s-q\times t-q}$. Jam sit $p+z^2\times q+z^2\times r+z^2\times s+z^2\times t+z^2=\alpha+\beta z^2+\gamma z^{22}+\delta z^{33}+\varepsilon z^{44}+z^{55}$; deinde erit $\varepsilon=p+q+r+s+t$, & $r+s+t=\varepsilon-w$, propterea quod est $p+q=w$; erit quoque $\delta=pq+pr+ps+pt+qr+qs+qt+rs+rt+st$, si igitur $r+s+t$ vel $\varepsilon-w$ appellemus A, cum sit $pq=v$, & $pr+ps+pt+qr+qs+qt=p+q\times r+s+t$, erit $rs+rt+st=\delta-v-Aw$; item erit $\gamma=pqr+pqs+pqt\ (=Av)+prs+prt+pst+qrs+qrt+qst+rst$; si igitur pro $rs+rt+st$ vel $\delta-v-Aw$ scribamus B, erit $rst=\gamma-Av-Bw$, quæ quantitas dicatur C. Porro est $r-p\times r-q=rr-r\times p+q+pq=rr-wr+v$; $s-p\times s-q=ss-ws+v$, & $t-p\times t-q=tt-wt+v$. Hinc tandem erit $kq+lp\ (=a)=\frac{C\times q-p-B\times qq-pp+A\times q^3-p^3-q^4-p^4}{q-p\times v-wr+rr\times v-ws+ss\times v-wt+tt}$
$=\frac{C-B\times q+p+A\times qq+qp+pp-q^3+qqp+qpp+p^3}{v-wr+rr\times v-ws+ss\times v-wt+tt}$;

$\&\ k+l\,(=b) = \frac{-B\times\overline{q-p}+A\times\overline{qq-pp}-\overline{|q^3-p^3}}{q-p\times\overline{v-wr+rr}\times\overline{v-ws+ss}\times v-wt+tt}$

$= \frac{-B+A\times\overline{q+p}-\overline{|qq+qp+pp}}{v-wr+rr\times\overline{v-ws+ss}\times v-wt+tt}$. Jam nihil amplius reſtat, niſi ut inveniamus quantitates $q+p$, $qq+qp+pp$, $q^3+qqp+qpp+p^3$; datur autem $q+p$; ſed data ſumma duarum quantitatum una cum rectangulo ſub ipſis, dico ſummam quoque dari omnium homogeneorum, quæ ab iis fiunt, uniuſcujuſque ordinis: verbi cauſa eſt $qp=v$; unde cum $q+p$ ſit $=w$, erit $qq+qp+pp=qq+2qp+pp-qp=ww-v$, item $q^3+qqp+qpp+p^3=q^3+3qqp+3qpp+p^3-2qp\times\overline{q+p}=w^3-2vw$, erit etiam $q^4+q^3p+qqpp+qp^3+p^4=q^4+4q^3p+6qqpp+4qp^3+p^4-3qp\times\overline{qq+2qp+pp}+qqpp=w^4-3vw^2+v^2$, & eodem modo proceditur in infinitum, ſunt autem quantitates w, $ww-v$, w^3-2vw, $w^4-3vw^2+v^2$, ſi pro v ſcribas 1, coefficientes terminorum, qui oriuntur dividendo unitatem per trinomium $1-wx+xx$; ideoque datur lex generalis, qua formantur *. Hac ratione invenimus a & b, eſt enim $a = \frac{C-Bw+A\times\overline{ww-v}-\overline{|w^3-2vw}}{v-wr+rr\times\overline{v-ws+ss}\times v-wt+tt}$; & $b = \frac{-B+Aw-\overline{|ww-v}}{v-wr+rr\times\overline{v-ws+ss}\times v-wt+tt}$. Quantitates m, n, o faciliùs inveniuntur; cum enim ſit $m = \frac{1}{\overline{p-r\times q-r}\times\overline{s-r\times t-r}}$; & $\overline{p-r\times q-r} = pq-\overline{|p+q\times r+rr} = v-wr+rr$, erit $m = \frac{1}{v-wr+rr\times\overline{s-r\times t-r}}$. Eodem modo eſt $n = \frac{1}{v-ws+ss\times\overline{r-s\times t-s}}$; & $o = \frac{1}{v-wt+tt\times\overline{r-t\times s-t}}$.

Præterea, ſi ponatur $\overline{r+z'}\times\overline{s+z'} = \zeta+\xi z'+z''$, ut ſit $\zeta = rs$, & $\xi = r+s$, & deinde ponatur $\frac{1}{v+wz'+z''\times\zeta+\xi z'+z''\times t+z'}$

* Vid. Pag. 46.

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$= \frac{a + bx^{\nu}}{v + wx^{\nu} + x^{2\nu}} + \frac{\pi + \varrho x^{\nu}}{\zeta + \xi x^{\nu} + x^{2\nu}} + \frac{o}{\varsigma + x^{\nu}}$, inveniri poſſunt a, b, π, ϱ, o; ponendo enim $v - \zeta = \sigma$, & $w - \xi = \tau$, erit $\overline{v - wr + rr} \times \overline{v - ws + ss}$ $(= vv - vwr + vrr + wwrs - wrrs + rrss = - vws + vss \quad - wrss$

$vv - vw\xi + v\xi\xi - 2v\zeta + ww\zeta - w\zeta\xi + \zeta\zeta) = \sigma\sigma - \tau \times \overline{v\xi - \zeta w}$;

ideoque a $(= \frac{C - Bw + A \times \overline{ww - v} - \sqrt{w^3 - 2vw}}{v - wr + rr \times v - ws + ss \times v - wt + tt})$ eſt

$= \frac{C - Bw + A \times \overline{w\,w - v} - \sqrt{w^3 - 2vw}}{\sigma\sigma - \tau \times v\xi - \zeta w \times v - wt + tt}$, & b $(=$

$\frac{-B + Aw - \sqrt{ww - v}}{v - wr + rr \times v - ws + ss \times v - wt + tt}) = \frac{-B + Aw - \sqrt{ww - v}}{\sigma\sigma - \tau \times v\xi - \zeta w \times v - wt + tt}$.

Eadem ratione ponendo $\zeta - v = \varphi$, & $\xi - w = \chi$, item $\varsigma - \xi = H$, $\delta - \zeta - H\xi = I$, & $\gamma - H\zeta - I\xi = K$, erit $\pi = \frac{K - I\xi + H \times \xi\xi - \zeta - \sqrt{\xi^3 - 2\zeta\xi}}{\varphi\varphi - \chi \times \zeta w - v\xi \times \zeta - \xi t + tt}$, & $\varrho = \frac{-I + H\xi - \sqrt{\xi\xi - \zeta}}{\varphi\varphi - \chi \times \zeta w - v\xi \times \zeta - \xi t + tt}$:

eſt autem o $(= \frac{1}{v - wt + tt \times r - t \times s - t}) = \frac{1}{v - wt + tt \times \zeta - \xi t + tt}$

Eodem modo conjungere licet tres vel plures ex fractionibus binomiis; quandocunque autem contingit aliquas earum fractionum denominatores habere eoſdem, omnes illæ in unam hac ratione ſunt contrahendæ; idemque quando fractiones trinomiæ eoſdem habent denominatores faciendum. Vel etiam curvam in his caſibus metiri licet ejiciendo e denominatore omnes præter unum factorum, qui ſunt inter ſe æquales, ordinatamque reducendo ad fractionem, cujus denominator ſit idonea poteſtas quantitatis inde oriundæ.

Si multinomio ſex adſint termini, ponendo illud $= o$, formatur æquatio ad quinque dimenſiones aſcendens, quæ ad minimum unam habebit radicem, habere autem poteſt aliquando tres vel etiam quinque, & proinde multinomium reduci poteſt, vel in factores quinque binomios, vel in tres binomios cum uno trinomio factore, vel ſaltem in unum factorem binomium & in quinquinomium; qui quinquinomius factor, ut jam oſtendi, ſemper reduci poteſt in duos trinomios. Hac autem reductione peracta, facile eſt ex præmiſſis ordinatam propoſitam in ordinatas cum binomiis & trinomiis denominatoribus reducere.

Porro harum ordinatarum denominatores ex pluribus quam sex terminis componi possunt; sed, quæ sit ratio commodissima ejusmodi denominatores in trinomia sua resolvendi, quando in binomia reduci nequeunt, accurate inquirere otium in præsentia non est. Hoc igitur negotium tibi, vir amicissime, commendo; in quem finem utile forsitan erit cognoscere, quod si trinomium $x + yz^{\nu} + z^{2\nu}$ quemcunque denominatorem dividit hujus formæ $e + fz^{\nu} + gz^{2\nu} + hz^{3\nu} + kz^{4\nu} + lz^{5\nu} + mz^{6\nu} + nz^{7\nu} + pz^{8\nu}$ + &c, termini x & y his duabus æquationibus inveniuntur,

$$e = \begin{cases} gx - hyx + ky^2x - ly^3x + my^4x - ny^5x + py^6x - \&c \\ + hx^2 - 2kyx^2 + 3ly^2x^2 - 4my^3x^2 + 5ny^4x^2 - \&c \\ + mx^3 - 3nyx^3 + 6py^2x^3 - \&c \\ - px^4 + \&c \\ + \&c \end{cases}$$

$$0 = \begin{cases} f - gy + hy^2 - ky^3 + ly^4 - my^5 + ny^6 - py^7 + \&c \\ - hx + 2kyx - 3ly^2x + 4my^3x - 5ny^4x + 6py^5x - \&c \\ + lx^2 - 3myx^2 + 6ny^2x^2 - 10py^3x^2 + \&c \\ - nx^3 + 4pyx^2 - \&c \\ + \&c \end{cases}$$

In his autem utrisque æquationibus numeri figurati earum terminis præfiguntur.

Hactenus *Cotesium* secuti sumus per omnia, quæ tradidit, ad ordinatas rationales pertinentia, jam verbum unum aut alterum de irrationalibus addetur. Harum curvarum perspicacissimus vir quadraturam dedit eas reducendo ad rationales. Si ordinata sit $dz^{b-1} \times \overline{e + fz^{\nu}}|^{\frac{\theta}{\lambda}}$, curvæ area æqualis est areæ curvæ, cujus ordinata rationalis est, quandocunque θ est numerus integer seu affirmativus seu negativus, aut quando est $= 0$; ut apparebit ponendo $e + fz^{\nu} = x^{\nu}$; nam hoc pacto area curvæ, cujus abscissa est z & ordinata $dz^{b-1} \times \overline{e + fz^{\nu}}|^{\frac{\theta}{\lambda}}$, invenietur æqualis areæ curvæ, cujus abscissa est x & ordinata $\frac{vd}{nf} x^{v+\frac{t}{\lambda}-1} \times \overline{\frac{-e+x^{\nu}}{f}}|^{\frac{\theta}{\nu}-1}$. Et etiam area curvæ, cujus ordinata est $dz^{b-1} \times \overline{e + fz^{\nu} + gz^{2\nu} + hz^{3\nu} + \&c}|^{\frac{\pi}{\nu}} \times \overline{k + lz^{\nu}}|^{\frac{\theta}{\lambda}}$, si θ idem quod antea denotet, & π quemvis numerum integrum seu affirmativum seu negativum, reducitur ad aream curvæ, cujus ordinata rationalis

tionalis est: nam ponendo $k+lx^{\nu}=x^{\nu}$, ea ordinata rationalis erit $\frac{\nu d}{\nu f}x^{\nu+\frac{\varepsilon}{\lambda}-\nu}\times e+f\times\frac{-k+x^{\nu}}{l}+g\times\frac{-k+x^{\nu}}{l}\Big|^{2}+b\times\frac{-k+x^{\nu}}{l}\Big|^{3}+\&c\Big|^{\theta}\times\frac{-k+x^{\nu}}{l}\Big|^{\theta-1}$. Si autem ponatur $\theta=1$, prior ordinata rationalis fiet $=\frac{\nu d}{\nu f}x^{\nu+\frac{\varepsilon}{\lambda}-\nu}$, & posterior $=\frac{\nu d}{\nu f}x^{\nu+\frac{\varepsilon}{\lambda}-\nu}\times e+f\times\frac{-k+x^{\nu}}{l}+g\times\frac{-k+x^{\nu}}{l}\Big|^{2}+b\times\frac{-k+x^{\nu}}{l}\Big|^{3}$: denique si fit $\theta=0$ ordinata prior fiet $=\frac{x}{\eta}dx^{\eta+\frac{\varepsilon}{\lambda}-\nu}\times\frac{1}{-e+x^{\nu}}$, & posterior $=\frac{x}{\eta}dx^{\eta+\frac{\varepsilon}{\lambda}-\nu}\times e+f\times\frac{-k+x^{\nu}}{l}+g\times\frac{-k+x^{\nu}}{l}\Big|^{2}+b\times\frac{-k+x^{\nu}}{l}\Big|^{3}\times\frac{1}{-e+x^{\nu}}$.

Porro area curvæ, cujus ordinata est $dx^{\eta-\frac{\varepsilon}{\lambda}\nu-1}\times e+fx^{\nu}\Big|^{\frac{\varepsilon}{\lambda}}$, æqualis est areæ curvæ, cujus ordinata rationalis est, ea lege ut θ sit $=0$, aut alicui numero integro affirmativo vel negativo; quoniam mutando signum indicis η, $dx^{\eta-\frac{\varepsilon}{\lambda}\nu-1}\times e+fx^{\nu}\Big|^{\frac{\varepsilon}{\lambda}}$ invenietur $=dx^{\eta-1}\times f+ex^{-\nu}\Big|^{\frac{\varepsilon}{\lambda}}$. Eodem modo ordinata $dx^{\eta-\frac{\varepsilon}{\lambda}\nu-1}\times e+fx^{\nu}+gx^{2\nu}\ldots+bx^{\mu\nu}\Big|^{\varepsilon}\times k+lx^{\nu}\Big|^{\frac{\varepsilon}{\lambda}}$, mutando signum η, fiet $dx^{\nu\mu+\eta-1}\times b\ldots+gx^{-\nu\times\mu-2}+fx^{-\nu\times\mu-1}+ex^{-\nu\mu}\Big|^{\varepsilon}\times l+kx^{-\nu}\Big|^{\frac{\varepsilon}{\lambda}}$.

Præterea curva, cujus ordinata est $dx^{\eta-1}\times\frac{e+fx^{\nu}}{g+hx^{\nu}}\Big|^{\frac{\varepsilon}{\lambda}}$, mensuram recipit a curva, cujus ordinata rationalis est; nam ponendo $g+hx^{\nu}=x^{\nu}$, curva, cujus ordinata est $dx^{\eta-1}\times\frac{e+fx^{\nu}}{g+hx^{\nu}}\Big|^{\frac{\varepsilon}{\lambda}}$, invenietur æqualis curvæ, cujus ordinata est $\frac{\nu d}{\eta b}x^{\eta-\frac{\varepsilon}{\lambda}\nu-1}\times\frac{-g+x^{\nu}}{b}\Big|^{\theta-1}\times$

$\times \overline{\frac{eb-fg}{b}+\frac{f}{b}x^{r}}\Big|^{\frac{s}{\lambda}}$ (vel mutando ſignum r) $=\frac{\nu d}{qb}x^{\frac{\nu}{q}-1}\times$

$\overline{1-\frac{gx^{-r}}{b}}\Big|^{\frac{\nu}{q}-1}\times\overline{\frac{f}{b}+\frac{eb-fg}{b}x^{-r}}\Big|^{\frac{s}{\lambda}}$. Et eodem modo curva, cujus

ordinata eſt $dz^{\frac{\nu}{q}-1}\times\overline{k+lz^{r}+mz^{2r}+nz^{3r}+\&c}\Big|^{\pi}\times\overline{\frac{e+fz^{r}}{g+bz^{r}}}\Big|^{\frac{s}{\lambda}}$, π

exiſtente numero integro, reducitur ad curvam, cujus ordinata rationalis eſt.

Et hæc ſummatim continent omnia, quæ magnus noſter geometra docuit de ordinatis, quæ ex binomiis irrationalibus componuntur. Aggreditur quoque trinomia irrationalia, & non ſolum eas curvas, quas ſummus *Newtonus* prius conſideravit, ſuo modo tractat; ſed confert quoque cum circulo & hyperbola curvas, quarum ordinatæ ſunt

$\frac{dz^{\frac{\nu}{q}-1}}{k+lz^{r}\sqrt{e+fz^{r}+gz^{2r}}}$, $\frac{dz^{\frac{\nu}{q}-1}\sqrt{e+fz^{r}+gz^{2r}}}{k+lz^{r}}$, & etiam

$\frac{dz^{\frac{\nu}{q}-1}}{k+lz^{r}+mz^{2r}\sqrt{e+fz^{r}+gz^{2r}}}$, $\frac{dz^{\frac{\nu}{q}-1}\sqrt{e+fz^{r}+gz^{2r}}}{k+lz^{r}+mz^{2r}}$. Harum duæ priores perfecte & ſine limitatione menſuram habent eodem modo, quo in prioribus tabulis illas dimetiri diximus. Duarum ultimarum areæ eo tantum caſu exhibentur, quando trinomium $k+lz^{r}+mz^{2r}$ reſolvi poteſt in duos factores binomios: & profecto ita dividendo hoc trinomium hæ curvæ ad præcedentes referuntur.

Curvæ autem iſtæ, quarum ordinatæ complectuntur trinomia irrationalia, tractari poſſunt ſine limitatione illas reducendo ad curvas, ordinatam habentes, cujus pars irrationalis non niſi binomium eſt, idque non tantum cum pars rationalis denominatoris eſt trinomium, ſed quando ea pars multinomium eſt quodcunque, ut jam tibi oſtendam. In quem finem has curvas ad duos caſus reducam, quorum primus eſt, quando terminus medius trinomii irrationalis abeſt. Verbi cauſa ſit ordinata $\frac{dz^{\frac{\nu}{q}-1}}{k+lz^{r}+mz^{2r}+nz^{3r}+\&c\times\sqrt{e+gz^{2r}}}$, vel

$\frac{dz^{\frac{\nu}{q}-1}\sqrt{e+gz^{2r}}}{k+lz^{r}+mz^{2r}+nz^{3r}+\&c}$, quæ ordinatæ propter impares dimenſiones quantitatis z in partibus rationalibus, ſub iis, quæ ſupra de ordinatis

natis binomiis ſcripta ſunt, non cadunt. Metiuntur autem curvæ, quarum hæ ſunt ordinatæ, auferendo impares iſtas poteſtates ex denominatoribus; quod fit multiplicando ordinatas per $k - lz^{\nu} + mz^{2\nu} - nz^{3\nu} + \&c$; unde invenietur ordinata prior $= \frac{dkz^{\lambda-1} - dlz^{\lambda+\nu-1} + dmz^{\lambda+2\nu-1} - dnz^{\lambda+3\nu-1} + \&c}{kk + 2km - ll \times z^{2\nu} + mm - 2ln \times z^{3\nu} - nnz^{4\nu} \&c} \times \sqrt{e + gz^{2\mu}}$; & ordinata altera $= \frac{dkz^{\lambda-1} - dlz^{\lambda+\nu-1} + dmz^{\lambda+2\nu-1} - dnz^{\lambda+3\nu-1} + \&c \times \sqrt{e + gz^{2\mu}}}{kk + 2km - ll \times z^{2\nu} + mm - 2ln \times z^{3\nu} - nnz^{4\nu} + \&c}$.

Hæ autem ultimæ ordinatæ, ut includunt irrationalem ex duobus terminis conſtantem, ita ad curvas pertinent, quæ antea ſunt tractatæ; nam ſi β ſit $= 0$, aut denotet numerum quemvis integrum affirmativum vel negativum, cum $\frac{\beta}{\lambda}$ hinc ſit $= \pm \frac{1}{1}$, manifeſtum eſt ſingula membra harum ordinatarum comprehenſa eſſe ſub altero ex duobus caſibus ante memoratis binomiorum irrationalium ſimplicium.

Si trinomium irrationale omnibus ſuis terminis ſit completum, ponendo $\frac{1}{4}f + gz^{\nu} = x^{\nu}$ invenietur $g \times e + fx^{\nu} + gx^{2\nu} = eg - \frac{1}{4}ff + x^{2\nu}$ trinomio medio ſuo termino carenti; adeo ut hoc pacto caſus præſens ad præcedentem reducitur.

Observabis autem in omnibus, quas hic tractavi, curvis ordinatarum denominatores vel omnino rationales eſſe, vel unica quantitate irrationali conſtare, vel denique irrationali aliqua quantitate in rationalem ducta: poſſunt tamen ordinatarum denominatores a ſumma vel differentia quantitatis rationalis & irrationalis vel duarum irrationalium componi. Has autem ordinatas ad alias denominatorum rationalium reducere licet, ſive quantitates illæ irrationales in rationales ducantur, ſive non. Verbi cauſa ſit $e + fz^{\nu} + \&c = R$, $g + bz^{\nu} + \&c = S$, $k + lz^{\nu} + \&c = T$, $m + nz^{\nu} + \&c = V$; & ſit ordinatæ alicujus denominator $= RS^{\frac{\lambda}{\xi}} \pm TV^{\frac{\mu}{\nu}}$, ubi λ, ξ, μ, ν numeros integros denotent. Numerorum λ, μ ſit ζ maxima communis menſura; ſit autem $\frac{\lambda}{\zeta} = \varkappa$, & $\frac{\mu}{\zeta} = \xi$, adeo ut $\lambda\xi$ ſit $= \mu\varkappa$. Jam vero erit $RS^{\frac{\lambda}{\xi}} - TV^{\frac{\mu}{\nu}} \times R^{\varkappa-1}S^{\varkappa-\frac{\lambda}{\xi}} + R^{\varkappa-2}S^{\varkappa-\frac{2\lambda}{\xi}}TV^{\frac{\mu}{\nu}} + R^{\varkappa-3}S^{\varkappa-\frac{3\lambda}{\xi}}T^2V^{\frac{2\mu}{\nu}}$

$+ \&c. \ldots + T^{\mu-1} V^{\frac{\nu}{\mu}\times\mu-1} = R^{\lambda\xi} S^{\nu\xi} - T^{\mu} V^{\nu}$. Item $RS^{\frac{\delta}{\lambda}} + TV^{\frac{\varepsilon}{\mu}} \times R^{\lambda\xi-1} S^{\nu\xi-\frac{\delta}{\lambda}} - R^{\lambda\xi-2} S^{\nu\xi-\frac{2\delta}{\lambda}} TV^{\frac{\varepsilon}{\mu}} + R^{\lambda\xi-2} S^{\nu\xi-\frac{2\delta}{\lambda}} T^2 V^{\frac{2\varepsilon}{\mu}} - \&c. \ldots \pm T^{\mu-1} V^{\frac{\nu}{\mu}\times\mu-1}$, si $\lambda\xi$ vel $\mu\nu$ sit numerus impar, erit $= R^{\lambda\xi} S^{\nu\xi} + T^{\mu\nu} V^{\mu\nu}$; si vero $\lambda\xi$ vel $\mu\nu$ sit numerus par, erit idem factum $= R^{\lambda\xi} S^{\nu\xi} - T^{\mu\nu} V^{\mu\nu}$ *. Hinc, si ducatur ordinata proposita in $R^{\lambda\xi-1} S^{\nu\xi-\frac{\delta}{\lambda}} + R^{\lambda\xi-2} S^{\nu\xi-\frac{2\delta}{\lambda}} TV^{\frac{\varepsilon}{\mu}}$ &c, vel in $R^{\lambda\xi-1} S^{\nu\xi-\frac{\delta}{\lambda}} - R^{\lambda\xi-2} S^{\nu\xi-\frac{2\delta}{\lambda}} TV^{\frac{\varepsilon}{\mu}}$ &c, convertetur denominator in quantitatem rationalem, nempe in hanc $R^{\lambda\xi} S^{\nu\xi} \pm T^{\mu\nu} V^{\mu\nu}$, omnibus quantitatibus irrationalibus in ordinatæ numeratorem translatis.

Modo haud absimili res etiam perfici potuit, si ordinata denominatorem habuit $R^{\frac{\tau}{\iota}} S^{\frac{\delta}{\lambda}} \pm T^{\frac{\varepsilon}{\tau}} V^{\frac{\varepsilon}{\mu}}$, ne ordinatas magis adhuc compositas memorem.

In hujus rei exemplum sumatur ordinata $\frac{dz^{\delta\nu-1}}{\sqrt{b+kz^{\delta}} \times \sqrt{e+fz^{\varepsilon}+gz^{2\varepsilon}} - \sqrt{p+qz^{\varepsilon}+rz^{2\varepsilon}} \sqrt{l+mz^{\varepsilon}}}$. Hic est $R = b + kz^{\delta}$, $S = e + fz^{\varepsilon} + gz^{2\varepsilon}$, $T = p + qz^{\varepsilon} + rz^{2\varepsilon}$, $V = l + mz^{\varepsilon}$, $\delta = 1 = \iota$, $\lambda = 2 = \mu$; item $\zeta = 2$, & $\nu = 1 = \xi$. Ideoque ducenda est ordinata in $\sqrt{b+kz^{\delta}} \times \sqrt{e+fz^{\varepsilon}+gz^{2\varepsilon}} - \sqrt{p+qz^{\varepsilon}+rz^{2\varepsilon}} \sqrt{l+mz^{\varepsilon}}$, unde perveniet $\frac{dbz^{\delta\nu-1} + dkz^{\delta\nu+\delta-1} \times \sqrt{e+fz^{\varepsilon}+gz^{2\varepsilon}} - \sqrt{dpz^{\delta\nu-1} + dqz^{\delta\nu+\varepsilon-1} + drz^{\delta\nu+2\varepsilon-1}} \times \sqrt{l+mz^{\varepsilon}}}{\sigma + \tau z^{\varepsilon} + \phi z^{2\varepsilon} + \chi z^{3\varepsilon} + \psi z^{4\varepsilon} + \omega z^{5\varepsilon}}$; ubi est $\sigma = ebb - lpp$, $\tau = 2ebk + fbb - 2lpq - mpp$, $\phi = ekk + 2fbk + gbb - 2lpr - lqq - 2mpr$, $\chi = fkk + 2gbk - 2lqr - 2mpr - mqq$, $\psi = gkk - lrr - 2mqr$, & $\omega = -mrr$.

Hic habes vir amicissime, præcipua, quæ hactenus de *Cotesii* admirabilibus inventis observavi. Nihil de theorematis dixi, quæ ad tabulas suas continuendas construxit, quia sine ulla singulari arte a propositionibus septima & octava *Quadrat. curv. Newtoni* deducuntur. Utilia autem admodum sunt hæc theoremata, quibus in solvendis problematis, supputandi tædium levari possit. Hic igitur finem prælongis

* Vid. *Wise* ad Logist. speciof. not. prior. Theorem. univers. post prop. 24.

his

his literis tandem imponam; ubi si res aliquando negligentius, quam oportuit, explicui, vel contra si quandoque copiosius exposui, quam necesse fuit ad te virum intelligentem, atque harum rerum peritissimum, scribens; pro veteri necessitudine qua olim fuimus conjuncti, & quæ indies increscit, scio te veniam amico familiariter loquenti facile daturum. Vale.

FINIS.

ERRATA.

PAG. 7. l. ult. dele vincul. prius in ordinat. denominat.
Pag. 11. in fig. poster. producenda fuit linea *AFN* donec lineis *CH*, *CL* etiam productis occurisset.
Pag. 14. l. ult. leg. $AEq \times \overline{-xx+2}$.
Pag. 23. l. penult. leg. $\overline{2AI+2AK+2AL} \times AP^{n-1}$.
Pag. 40. in marg. leg. de Moivre.
Pag. 42. l. 1, 2, 4, 5. del. vincul. super.
Pag. 42. l. 4. leg. $\sqrt[n]{\frac{z}{g}} + AE \times x^{\frac{1}{n}-1} + x^{\frac{2}{n}-1} \times$
Pag. 43. l. 2. dele *AB*.
Pag. 44. l. 6. leg. quantitatis.

his literis tradam impensam; ubi si res aliquando negligentiæ, quam
cæteris, culpam, vel contra si quandoque copiosius exposui, quam
necesse fuit ad te virum intelligentem, æque hanc rem ac me pertine-
mem, scribens; pro veteri necessitudine qua olim fuimus conjuncti,
& quæ indies increbuit, fac te veniam amico familiariter loquenti fa-
cile daturum. Vale.

FINIS.

ERRATA.

PAG. 7. l. ult. dele vinc. prius in ordinat. denominat.
Pag. 11. in fig. poster. producenda fuit linea MFN donec lineis
CH, CL esset prodeuntibus occurrisset.
Pag. 14. l. ult. leg. $\frac{AE^2}{4} \times -xx + z$.
Pag. 23. l. penult. leg. $z\,AI + zAK + zAL \times AP$.
Pag. 40. in marg. leg. de Moivre.
Pag. 42. l. 13. $z + 5$. del. vinc. super.
Pag. 42. l. 4. leg. $\sqrt{\frac{}{}} + AE \times z + zx \times$
Pag. 43. l. 2. dele $\frac{1}{2}B$.
Pag. 44. l. 6. leg quantitate.

APPENDIX
AD
EPISTOLAM
DE
COTESII Inventis,
CURVARUM RATIONE,
QUÆ
Cum CIRCULO & HYPERBOLA Comparationem admittunt.

EPISTOLAM ab amico ad me datam de iis, quæ clarissimus Cotesius *invenit curvarum ratione, quæ cum circulo & hyperbola comparationem admittunt, haud diu edideram, cum ab epistolæ istius auctore aliud scriptum accepi; quo rationem*

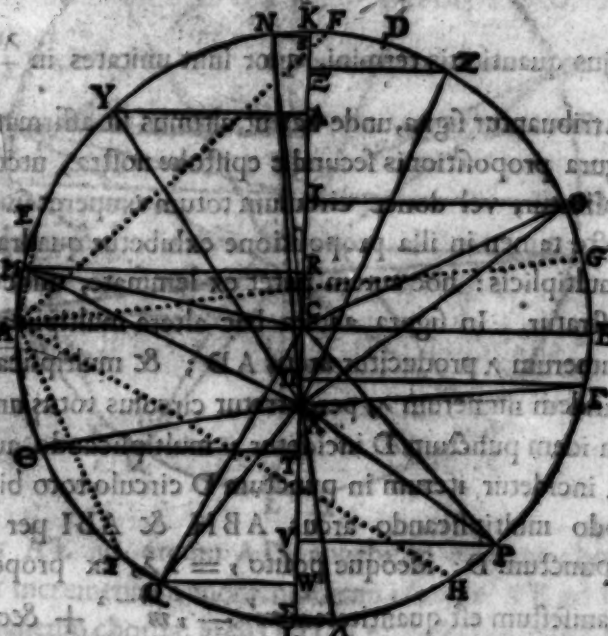
genera-

generalem trinomia in alia dividendi, curvasque inde metiendi, quarum ordinatæ fractiones sunt rationales cum denominatoribus trinomiis, in ea epistola traditam, ulterius persequitur. Cum istis igitur amici inventis forma hic tribuatur, ei haud absimilis, quam in curvis metiendis, quarum ordinatæ fractiones sunt binomiæ & rationales, Cotesius *ipse elegantissime adhibuit; scriptum haud indignum judicavi, quod lucem aspiceret; ideoque publici juris jam feci, ut epistolam comitaretur.*

J. W.

M 415. 17.
1723.

SIT trinomium dividendum $s = b x^{\kappa} + x^{\mu}$. Primum vero sit λ numerus impar. Radio unitari æquali describatur circulus, cujus diameter sit AB, centrum C. In hoc circulo sumatur arcus AD, cujus chorda æqualis sit b; & sit arcus AE ad arcum AD ut s ad λ; deinde dividatur circulus in tot partes æquales EF, FG, GH, HI, IE, quot sunt unitates in λ, ductis chordis AE, AF, AG, AH, AI. Porro ducatur diameter KL diametro AB normalis; &

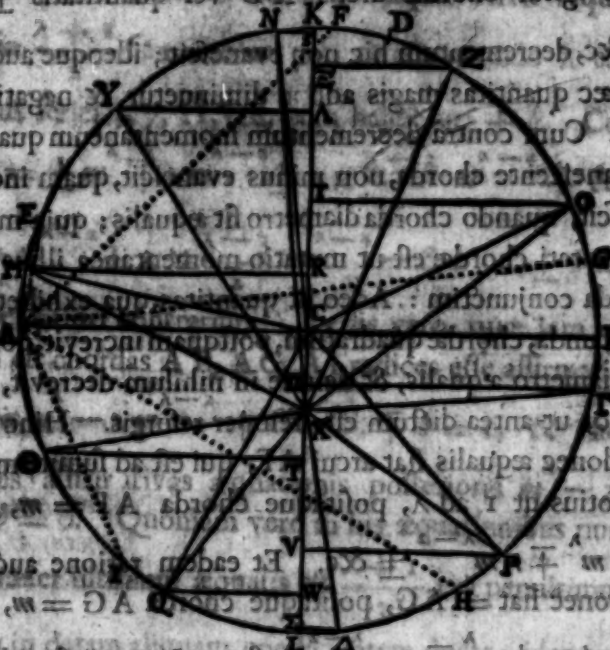


diviso arcu AE in partes duas æquales in M, ita ut AM sit ad AD ut 1 ad 2λ, dividatur circulus totus in tot partes æquales MN, NO, OP,

OP, PQ, QM, quot ſunt unitates in λ, dimiſſis ad KL perpendiculis MR, NS, OT, PV, QW. Deinde cum arcus AM dimidius ſit arcus AE, erit CR pars dimidia chordæ AE: erit autem & arcus AN dimidius arcûs AG, propterea quod arcus MN (= EF) dimidius eſt arcûs EG; ideoque CS pars dimidia eſt chordæ AG: item arcus AQ dimidius eſt arcûs AOI, propterea quod MO (= EG) pars dimidia eſt arcûs EOI; eſt igitur CT pars dimidia chordæ AI: Præterea arcus AQ dimidius eſt arcûs AH, quoniam MQ (= EI) pars eſt dimidia arcûs EH; unde CW pars dimidia eſt chordæ AH: denique arcûs AP dimidius eſt arcûs APF, quia MP (= EH) dimidia pars eſt arcûs EHF; eſt igitur & CV dimidia pars chordæ AF. Jam ſi ponatur AE = m, in epiſtola * obſervatur chordam λ eſſe æqualem $\pm m \mp \lambda m \pm \lambda \times \frac{\lambda-1}{2} m \mp$ &c. Si modo tot ſumantur hujus quantitatis termini, quot ſunt unitates in $\frac{\lambda+1}{2}$, & iſtis terminis ea tribuantur ſigna, unde fiat ut ultimus ſit affirmativus. Præterea in figura propoſitionis ſecundæ epiſtolæ noſtræ, utcunque arcus AB multiplicetur, vel donec circulum totum exuperet ſive ſemel ſive pluries, recte tamen in illa propoſitione exhibetur quadratum chordæ arcus multiplicis: hoc autem patet ex lemmate, unde propoſitio iſta demonſtratur. In figura autem hac altera multiplicando arcum AE per numerum λ producitur arcus AD; & multiplicando arcum AF per eundem numerum λ, percurretur circulus totus una cum arcu AD, & in idem punctum D incidetur; multiplicando autem arcum AG per λ incidetur iterum in punctum D circulo toto bis percurſo; eodem modo multiplicando arcus ABH & ABI per λ incidetur ſemper in punctum D: ideoque poſito $\nu = 2\lambda$, ex propoſitione iſta ſecunda manifeſtum eſt quantitatem $m^{\nu} - \nu m^{\nu-2}$ + &c, æqualem eſſe quadrato chordæ AD, cum $\frac{\nu}{2}$ vel λ ſit numerus impar, utrum m accipiatur pro chorda AE, an pro chorda AF, pro AG, pro AH,

* Coroll. 1. prop. 1. pag. 16, 17.

AH, an denique pro AI, & si ν vel λ numerus fuisset par, quantitas $-m^{\nu} + \nu m^{\nu-2} -$ &c. sub iisdem omnino conditionibus æquaret quadratum chordæ AD. Jam vero dico quantitatem ante positam $\pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c, cujus ultimus terminus sit affirmativus, æqualem esse ipsi chordæ arcus AD, si m denotet aliquam ex chordis AE, AG, AI; sed si aliquam denotet ex chordis AF, AH, hæc quantitas chordam arcus AD exhibebit sub signo negativo. Si arcus AD semicirculo minor sit, & gradatim augeatur chorda AE, manente



ratione arcus AE ad arcum AD, augebitur quoque chorda arcus AD; & erit incrementum momentaneum chordæ AE ad incrementum momentaneum chordæ arcûs AD in ratione composita ex ratione chordæ arcûs EB ad chordam arcûs BD, & ex ratione 1 ad λ. Sed quam primum arcus AE tantum augetur, ut sit ad semicirculum ut 1 ad λ, quantitas $\pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c, diametro sit æqualis, &

desinet

defunctæ augeri, incremento ejus momentaneo evanescente, ita ut aucto amplius arcu AE decrescit quantitas $\pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c, æqualis chordæ arcûs AD; donec tandem in nihilum redigatur: quod postremum tum fit, quando arcus AE est ad totam circuli circumferentiam ut 1 ad λ. Sed ultima ratio incrementi momentanei chordæ AE, quod gignitur dum chorda arcûs AD ad circumferentiam totam evanescit, ad chordam evanescentem componitur ex ratione chordæ arcûs EB ad circuli diametrum &c ex ratione 1 ad λ. Momentaneum igitur chordæ arcûs AD vel quantitatis $\pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c, decrementum hic non evanescit; ideoque aucto insuper arcu AE hæc quantitas magis adhuc diminuetur, & negativa ex affirmativa fiet. Cum contra decrementum momentaneum quadrati hujus chordæ, evanescente chorda, non minus evanescit, quam incrementum ejus evanescit, quando chorda diametro fit æqualis; quia momentanea mutatio quadrati chordæ est ut mutatio momentanea illius chordæ, & ut ipsa chorda conjunctim: Adeo ut quantitas, qua exhibetur, ex propositione secunda, chordæ quadratum, postquam increvit, donec chorda fit circuli diametro æqualis, & deinde in nihilum decrevit, iterum sub eodem signo, ut antea dictum est, semper resurgit. Hinc vero aucto arcu AE, donec æqualis fiat arcui AF, qui est ad totum arcum AD & circuli totius ut 1 ad λ, positaque chorda AF $= m$, habebimus $-b = \pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c. Et eadem ratione aucto amplius arcu AE, donec fiat $= \frac{1}{2}$AG, positaque chorda AG $= m$, rursus habebimus $+b = \pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c. Item posita chorda AH $= m$, iterum inveniemus $-b = \pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c. Et denique si fit chorda AI $= m$, erit tertia vice $+b = \pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c. Scilicet si tot ubique quantitatis $\pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c termini,

quot sunt unitates in $\frac{\lambda+1}{2}$, ita sumantur, ut ultimus terminus sit affirmativus. Posita autem $b = \pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm \lambda \times \frac{\lambda-3}{2} m^{\lambda-4}$ $\mp$ &c, sub conditione memorata erit $m^{\lambda} - \lambda m^{\lambda-2} + \lambda \times \frac{\lambda-3}{2} m^{\lambda-4}$ — &c. . . . $\mp b = 0$, habebitur nempe $-b$, si $\frac{\lambda+1}{2}$ sit numerus impar; id est, si λ sit numerus aliquis seriei sequentis 5, 9, 13, &c: sin $\frac{\lambda+1}{2}$ sit numerus par, ut est, quando λ est numerus aliquis hujus seriei 3, 7, 11, &c, tum habebitur $+b$. Contra quando est $-b = \pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm \lambda \times \frac{\lambda-3}{2} m^{\lambda-4} \mp$ &c, habebitur $m^{\lambda} - \lambda m^{\lambda-2} + \lambda \times \frac{\lambda-3}{2} m^{\lambda-4}$ — &c. . . . $\pm b = 0$, ubi b signum habet priori contrarium. Ex iis vero, quæ jam ostendimus, manifestum est chordas A E, A G, A I radices esse affirmativas æquationis prioris $m^{\lambda} - \lambda m^{\lambda-2} +$ &c. . . . $\mp b = 0$, & chordas A F, A H radices affirmativas æquationis posterioris $m^{\lambda} - \lambda m^{\lambda-2} +$ &c. . . . $\pm b = 0$. Quoniam vero in his æquationibus numerus terminorum præter ultimum æqualis est $\frac{\lambda+1}{2}$, erit penultimus terminus ipsa radix m in datam aliquam coefficientem ducta; ideoque terminus ultimus b solus est, qui locum aliquem parem in his æquationibus occupat; igitur mutatio signi hujus termini mutabit duntaxat signum radicum; ideoque æquationis $m^{\lambda} - \lambda m^{\lambda-2} +$ &c. . . $\mp b = 0$ chordæ A E, A G, A I radices erunt affirmativæ, & A F, A H negativæ, si ponatur $-b$, quando λ aliquis est ex numeris 5, 9, 13, &c; & $+b$, quando λ aliquis est ex numeris 3, 7, 11, &c; sed chordæ A F, A H

radices

radices erunt affirmativæ, & AE, AG, AI negativæ ejusdem æquationis, si termino b tribuatur signum contrarium. Quoniam igitur ostendimus esse $2CR = AE$, $2CS = AG$, $2CT = AI$, item $2CW = AH$, $2CV = AF$; erunt quoque $2CR$, $2CS$, $2CT$ radices affirmativæ, & $2CV$, $2CW$ radices negativæ æquationis $m^{\lambda} - \lambda m^{\lambda-2} + \&c \mp b = 0$; si habetur $-b$, quando λ aliquis est ex numeris 5, 9, 13, &c; vel $+b$ quando λ aliquis est ex numeris 3, 7, 11, &c; & hæ radices eidem æquationi convenient signis mutatis, si habetur $+b$, quando λ aliquis est e numeris prioribus, vel $-b$, quando λ est aliquis e numeris posterioribus.

Hinc autem ex iis, quæ in epistola ostendimus, * intelligetur esse $\overline{1+2CR\times x+xx}\times\overline{1+2CS\times x+xx}\times\overline{1+2CT\times x+xx}\times\overline{1-2CV\times x+xx}\times\overline{1-2CW\times x+xx} = 1+bx^{\lambda}+x^{2\lambda}$, quando λ aliquis est e numeris 5, 9, 13, &c; & $= 1-bx^{\lambda}+x^{2\lambda}$, quando λ aliquis est e numeris 3, 7, 11, &c. Item esse $\overline{1-2CR\times x+xx}\times\overline{1-2CS\times x+xx}\times\overline{1-2CT\times x+xx}\times\overline{1+2CV\times x+xx}\times\overline{1+2CW\times x+xx} = 1+bx^{\lambda}+x^{2\lambda}$, quando λ aliquis est e numeris 3, 7, 11, &c, & $= 1-bx^{\lambda}+x^{2\lambda}$, quando λ aliquis est e numeris 5, 9, 13, &c.

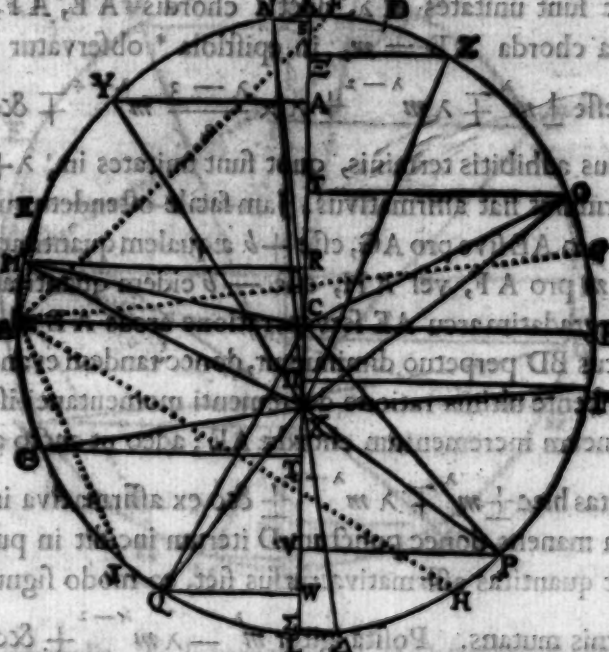
Porro sumatur $CX = x$, & ducantur XM, XN, XO, XP, XQ; item ex circuli centro CM, CN, CO, CP, CQ. Deinde erit $XMq = CMq + CXq + 2CR\times CX = 1 + 2CR\times x + xx$, $XNq = CNq + XCq + 2CS\times CX = 1 + 2CS\times x + xx$, $XOq = COq + XCq + 2CT\times CX = 1 + 2CT\times x + xx$, $XPq = CPq + XCq - 2CV\times CX = 1 - 2CV\times x + xx$, & $XQq = CQq + XCq - 2CW\times CX = 1 - 2CW\times x + xx$. Hinc est $1 + bx^{\lambda} + x^{2\lambda} = XMq\times XNq\times XOq\times XPq\times XQq$; quando λ aliquis est e numeri 5, 9, 13, &c; sed $1 - bx^{\lambda} + x^{2\lambda} = XMq\times$

* Pag. 47.

XNq

$XN^q \times XO^q \times XP^q \times XQ^q$, quando λ aliquis est e numeris 3, 7, 11, &c.

Præterea dividantur arcus MN, NO, OP, PQ, QM in partes duas æquales in punctis Y, Z, Γ, Δ, Θ; unde fiet arcus AΘ ad arcum ABD ut 1 ad 2λ, & circulus totus dividetur in punctis Θ, Δ, Γ, Z, Y in tot partes æquales, quot sunt unitates in λ. Dimittantur quoque in KL perpendicula VA, ZΞ, ΓΠ, ΔΣ, ΘT. Deinde erit $C\Pi = CR$, $C\Sigma = CS$, $C\Upsilon = CT$, $C\Lambda = CV$, & $C\Xi = CW$.



Item erit ut antea $XY^q = 1 + 2C\Lambda \times x + xx$, $XZ^q = 1 + 2C\Xi \times x + xx$, $X\Gamma^q = 1 - 2C\Pi \times x + xx$, $X\Delta^q = 1 - 2C\Sigma \times x + xx$, & $X\Theta^q = 1 - 2CT \times x + xx$. Quoniam igitur est $\frac{1 - 2CR \times x + xx}{} \times \frac{1 - 2CS \times x + xx}{} \times \frac{1 - 2CT \times x + xx}{} \times \frac{1 + 2CV \times x + xx}{} \times \frac{1 + 2CW \times x + xx}{} = 1 + bx^{\lambda} + x^{2\lambda}$, quando λ aliquis est e numeris 3, 7, 11, &c, &c $= 1 - bx^{\lambda} + x^{2\lambda}$,

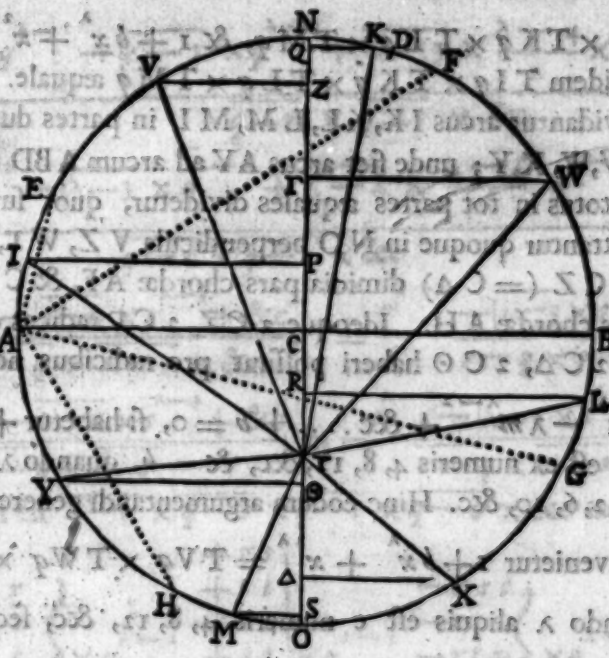
quando λ aliquis eſt e numeris 5, 9, 13, &c; erit $1 \mp b x^{\lambda} + x^{2\lambda} = XY\,q \times XZ\,q \times X\Gamma\,q \times X\Delta q \times X\Theta\,q$ in caſu priori, ſed $1 - b x^{\lambda} + x^{2\lambda} = XY\,q \times XZ\,q \times X\Gamma\,q \times X\Delta\,q \times X\Theta\,q$ in caſu poſteriori.

Hactenus numerus λ impar habitus eſt. Sit jam par; & deſcripto ut antea circulo, cujus diameter ſit AB, centrum C, in hoc circulo ſumatur arcus BD, cujus chorda ſit b; & ſit arcus AE ad AD ut 1 ad λ, dividaturque circulus in tot partes æquales EF, FG, GH, HE, quot ſunt unitates in λ, ductis chordis AE, AF, AG, AH. Jam poſita chorda $AE = m$, in epiſtlola * obſervatur chordam b æqualem eſſe $\pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm \lambda \times \frac{\lambda-3}{2} m^{\lambda-4} \mp$ &c, tot quantitatis hujus adhibitis terminis, quot ſunt unitates in $\frac{1}{2}\lambda + 1$, ſi modo ultimus terminus fiat affirmativus. Jam facile oſtendetur ut prius, ſive ponatur m pro AE ſive pro AG, eſſe $+b$ æqualem quantitati jam dictæ; ſed poſita m pro AF, vel AH, eſſe $-b$ eidem quantitati æqualem. Nam aucto gradatim arcu AE ſervata ratione arcus AE ad arcum AD, chorda arcus BD perpetuo diminuetur, donec tandem evaneſcit, non ſimul evaneſcente ultima ratione decrementi momentanei iſtius chordæ ad ſimultaneum incrementum chordæ AE; adeo ut aucto amplius arcu AE quantitas hæc $\pm m^{\lambda} \mp \lambda m^{\lambda-2} \pm$ &c ex affirmativa in negativam tranſit; ita manens donec punctum D iterum incidit in punctum B, & poſtea hæc quantitas affirmativa rurſus fiet, eo modo ſignum ſuum vicibus alternis mutans. Poſita igitur $m^{\lambda} - \lambda m^{\lambda-2} +$ &c $\ldots \mp b = 0$, ſi habetur $-b$, quando $\frac{1}{2}\lambda + 1$ numerus impar eſt, vel λ aliquis e numeris 4, 8, 12, &c, item ſi habetur $+b$, quando $\frac{1}{2}\lambda + 1$ numerus eſt par, vel λ aliquis e numeris 2, 6, 10, &c, erunt AE, AG radices affirmativæ hujus æquationis, & cum λ numerus ſit par, ita ut omnes poteſtates chordæ m in hac æquatione pares ſint, erunt eædem chordæ AE, AG negativæ quoque radices æquationis. Simi-

* Coroll. 2. prop. 2. pag. 17.

liter

liter, si habetur $+b$, quando λ aliquis est ex numeris 4, 8, 12, &c, vel $-b$, quando λ aliquis est ex numeris 2, 6, 10, &c, erunt AF, AH tum affirmative tum negative sumptæ hujus æquationis radices. Præterea si arcus AE in duas partes æquales dividatur in I, id est, si sumatur arcus AI ad AD ut 1 ad 2λ, & deinde si dividatur circuli circumferentia in tot partes æquales IK, KL, LM, MI, quot sunt unitates in λ, ducatur diameter NO, quæ diame-



tro AB sit normalis, & dimittantur in NO perpendicula IP, KQ, LR, MS, invenietur, ut antea, CP (=CR) dimidia pars chordæ AE, & CQ (=CS) dimidia pars chordæ AG; ideoque 2CP, 2CQ radices erunt affirmativæ, & 2CR, 2CS haberi possunt pro radicibus negativis æquationis $m^{\lambda} - \lambda m^{\lambda-2} + \&c. \mp b = 0$, si habetur $-b$, quando λ aliquis est e numeris 4, 8, 12, &c, vel $+b$, quando λ aliquis est e numeris 2, 6, 10, &c. Quum vero

hæc

hæc ita ſint, ex iis, quæ in epiſtola * oſtenſa ſunt, cognoſcetur $\overline{1+2CP\times x+xx}\times\overline{1+2CQ\times x+xx}\times\overline{1-2CR\times x+xx}\times\overline{1-2CS\times x+xx}$ &c. æquale eſſe $1-bx^{\lambda}+x^{2\lambda}$, quando λ aliquis eſt ex numeris 4, 8, 12, &c, ſed æquale $1+bx^{\lambda}+x^{2\lambda}$, quando λ aliquis eſt e numeris 2, 6, 10, &c. Unde ſi ſumatur $CT=x$, ducanturque TI, TK, TL, TM, erit $1-bx^{\lambda}+x^{2\lambda}$ in caſu primo æquale $TIq\times TKq\times TLq\times TMq$, & $1+bx^{\lambda}+x^{2\lambda}$ in caſu poſteriori eidem $TIq\times TKq\times TLq\times TMq$ æquale.

Porro dividantur arcus IK, KL, LM, MI in partes duas æquales in punctis V, W, X, Y; unde fiet arcus AY ad arcum ABD ut 1 ad 2λ, & circulus totus in tot partes æquales dividetur, quot ſunt unitates in λ. Dimittantur quoque in NO perpendicula VZ, WΓ, XΔ, YΘ. Deinde erit CZ (=CΔ) dimidia pars chordæ AF, & CΓ (=CΘ) dimidia pars chordæ AH. Ideoque $2CZ$, $2C\Gamma$ radices erunt affirmativæ, & $2C\Delta$, $2C\Theta$ haberi poſſunt pro radicibus negativis æquationis $m^{\lambda}-\lambda m^{\lambda-2}+$ &c. $\ldots\pm b=0$, ſi habetur $+b$, quando λ aliquis eſt ex numeris 4, 8, 12, &c, & $-b$, quando λ aliquis eſt ex numeris 2, 6, 10, &c. Hinc eodem argumentandi genere, quo prius ſum uſus, invenietur $1+bx^{\lambda}+x^{2\lambda}=TVq\times TWq\times TXq\times TYq$, quando λ aliquis eſt e numeris 4, 8, 12, &c, ſed $1-bx^{\lambda}+x^{2\lambda}=TVq\times TWq\times TXq\times TYq$, quando λ aliquis eſt ex numeris 2, 6, 10, &c.

Hac ratione reſolvitur omne trinomium, quando in binomia dividi non poteſt, in tot trinomia, quot requirantur, modo ei haud diſſimili, quo *Coteſius* binomium dividit: Hinc autem quædam iis adjici poſſunt, quæ prius de curvarum menſura ſcripſi, quarum ordinatis denominatores trinomii tribuuntur.

* Pag. 47.

Si

Si ea, quæ paginis 48, 49, epiſtolæ noſtræ habentur, cum pagina 46. ejuſdem conferantur; poſita ordinata $\frac{1}{1 \pm bx^{\lambda} + x^{\lambda+1}}$, vel $\frac{1}{1+qx+x^2} \times \frac{1}{1+rx+x^2} \times \frac{1}{1+sx+x^2} \times \&c = \frac{K+kx}{1+qx+xx} + \frac{L+lx}{1+rx+xx} + \frac{M+mx}{1+sx+xx} + \&c$, invenietur $K = \frac{q^{\lambda-1} - \overline{\lambda-2} \times q^{\lambda-3} + \overline{\lambda-3} \times \frac{\lambda-4}{2} \times q^{\lambda-5} - \&c}{\overline{q-r} \times \overline{q-s} \times \&c}$,

$k = \frac{q^{\lambda-2} - \overline{\lambda-3} \times q^{\lambda-4} + \overline{\lambda-4} \times \frac{\lambda-5}{2} q^{\lambda-6} - \&c}{\overline{q-r} \times \overline{q-s} \times \&c}$,

$L = \frac{r^{\lambda-1} - \overline{\lambda-2} \times r^{\lambda-3} + \&c}{\overline{r-q} \times \overline{r-s} \times \&c}$, $l = \frac{r^{\lambda-2} - \overline{\lambda-3} \times r^{\lambda-4} + \&c}{\overline{r-q} \times \overline{r-s} \times \&c}$,

$M = \frac{s^{\lambda-1} - \overline{\lambda-2} \times s^{\lambda-3} + \&c}{\overline{s-q} \times \overline{s-r}}$, $m = \frac{s^{\lambda-2} - \overline{\lambda-3} \times s^{\lambda-4} + \&c}{\overline{s-q} \times \overline{s-r} \times \&c}$,

&c. Jam eſt $\overline{q-r} \times \overline{q-s} \times \overline{q-t} \times \overline{q-v} \times \&c =$

$$q^{\lambda-1}\left\{\begin{matrix}-r\\-s\\-t\\-v\\-\&c\end{matrix}\right\} \times q^{\lambda-2}\left\{\begin{matrix}+rs\\+rt\\+rv\\+st\\+sv\\+tv\\+\&c\end{matrix}\right\} \times q^{\lambda-3}\left\{\begin{matrix}-rst\\-rsv\\-rtv\\-stv\\-\&c\end{matrix}\right\} \times q^{\lambda-4}\left\{\begin{matrix}+rstv\\+\&c\end{matrix}\right\}$$

$\times q^{\lambda-5} - \&c$. Porro in hoc caſu q, r, s, t, v, &c radices ſunt æquationis $m^{\lambda} - \lambda m^{\lambda-2} + \lambda \times \frac{\lambda-3}{2} m^{\lambda-4} - \&c \ldots \mp b = 0$. Ideoque erit $q+r+s+t+v+\&c = 0$, & $-r-s-t-v-\&c = q$; item $q \times r+s+t+v+\&c + rs+rt+rv+st+sv+tv$

$+tv+\&c=-\lambda$, & $rs+rt+rv+st+sv+tv+\&c=-\lambda+qq$; præterea $q\times rs+rt+rv+st+sv+tv+\&c+rst+rsv+rtv+stv+\&c=0$, & $rst+rsv+stv+\&c=\lambda q-q^3$; amplius $q\times rst+rsv+rtv+stv+\&c+rstv+\&c=\lambda\times\frac{\lambda-3}{2}$, & $rstv+\&c=\lambda\times\frac{\lambda-3}{2}-\lambda\times qq+q^4$. Hinc invenietur $\overline{q-r}\times\overline{q-s}\times\overline{q-t}\times\overline{q-v}\times\&c$

$$=\begin{cases}+q^{\lambda-1}\\+q^{\lambda-1}\\+q^{\lambda-1}-\lambda q^{\lambda-3}\\+q^{\lambda-1}-\lambda q^{\lambda-3}\\+q^{\lambda-1}-\lambda q^{\lambda-3}+\lambda\times\frac{\lambda-3}{2}q^{\lambda-5}\\+\&c\quad-\&c\quad+\&c\end{cases}$$

ubi $q^{\lambda-1}$ toties repetitur, quot sunt numero radices q, r, s, t, v, &c, id est, quot sunt unitates in λ; $\lambda q^{\lambda-3}$ toties repetitur, quot sunt unitates in $\lambda-2$; $\lambda\times\frac{\lambda-3}{2}q^{\lambda-5}$ toties repetitur, quot sunt unitates in $\lambda-4$: Ideoque est $\overline{q-r}\times\overline{q-s}\times\overline{q-t}\times\overline{q-v}\times\&c=\lambda q^{\lambda-1}-\lambda\times\overline{\lambda-2}\times q^{\lambda-3}+\lambda\times\frac{\lambda-3}{2}\times\overline{\lambda-4}\times q^{\lambda-5}-\&c$, vel $=\lambda\times q^{\lambda-1}-\overline{\lambda-2}\times q^{\lambda-3}+\overline{\lambda-3}\times\frac{\lambda-4}{2}\times q^{\lambda-5}-\&c$. Hinc autem inveniemus $K=\frac{1}{\lambda}$; item $k=\frac{1}{\lambda}$

$$\&c-\overline{\lambda-3}\times q^{\lambda-4}+\overline{\lambda-4}\times\frac{\lambda-5}{2}q^{\lambda-6}-\overline{\lambda-5}\times$$

$$\times\overline{q^{\lambda-1}-\overline{\lambda-2}\times q^{\lambda-3}+\overline{\lambda-3}\times\frac{\lambda-4}{2}q^{\lambda-5}-\overline{\lambda-4}\times}$$

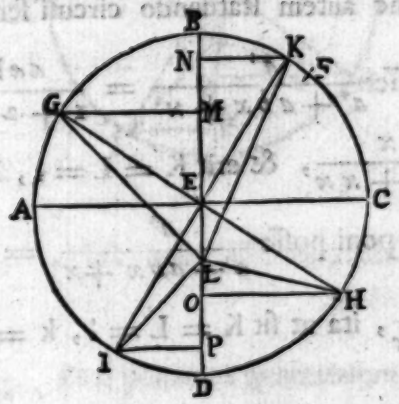
$\lambda-6$

$\frac{\lambda-6}{2}\times\frac{\lambda-7}{3}q^{\lambda-8}+\&c$
$\frac{\lambda-5}{2}\times\frac{\lambda-6}{3}q^{\lambda-7}+\&c$, ubi rejiciendi ſunt omnes termini in denominatore poſt iſtum terminum qui nihilo prodit æqualis, & numerus terminorum in numeratore ſumendorum uno termino minor eſſe debet quam in denominatore. Eadem ratione invenitur

$L=\frac{1}{\lambda}=M=\&c$, etiam $l=\frac{1}{\lambda}\times\frac{r^{\lambda-2}-\lambda-3\times r^{\lambda-4}+\&c}{r^{\lambda-1}-\lambda-2\times r^{\lambda-3}+\&c}$,

$\&c\ m=\frac{1}{\lambda}\times\frac{s^{\lambda-2}-\lambda-3\times s^{\lambda-4}+\&c}{s^{\lambda-1}-\lambda-2\times s^{\lambda-3}+\&c}$, &c.

Ut pauca exhibeam exempla, ſit primum curvæ metiendæ ordinata $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$. Ut hanc curvam ope eorum, quæ jam explicui, metiamur, primum dividatur trinomium $1\pm bx^n+x^{2n}$ in duo alia,



quod hac ratione fiet. Deſcribatur circulus ABCD ſemidiametro AE unitati æquali, ducatur diameter AC, & ſumatur arcus CF, cujus chorda ſit æqualis b, deinde ſumatur arcus AG quarta pars arcus AF, diviſo circulo toto in duas partes æquales in punctis G, H, ſi habeatur

habeatur $+b$, ſed ſi habeatur $-b$, ſumatur arcus A I quarta pars arcus A C F, & dividatur circulus totus in duas partes æquales in punctis I, K: deinde ducta diametro B E D ad perpendiculum diametro A C, ſumatur E L $= x$, ductis L G, L K, L H, L I; deinde erit $1 + bx^2 + x^4 = LGq \times LHq$, & $1 - bx^2 + x^4 = LIq \times LKq$. Hinc autem valde manifeſtum eſt, quod ſi circuli ſemidiameter non amplius pro unitate habeatur, ſed denotetur litera a, erit $a^4 + abx^2 + x^4 = LGq \times LHq$, & $a^4 - abx^2 + x^4 = LIq \times LKq$.

Porro demiſſis in BD perpendiculis G M, K N, H O, I P, ſi habeatur circuli ſemidiameter pro unitate, ex iis quæ noviſſime oſtenſa ſunt, poni poteſt $\frac{1}{1+bx^2+x^4} = \frac{K+kx}{1+2EM \times x + xx\,(=LGq)} + \frac{L+lx}{1-2EO \times x + xx\,(=LHq)}$, ubi K erit $= \frac{1}{2} = L$, k $= \frac{1}{4EM}$, l $= -\frac{1}{4EO}$, item erit $\frac{1}{1-bx^2+x^4} = \frac{K+kx}{1-2EP \times x + xx} + \frac{L+lx}{1+2EN \times x + xx}$, ſi K ponatur $= \frac{1}{2} = L$, k $= -\frac{1}{4EP}$, & l $= \frac{1}{4EN}$. Hinc autem ſtatuendo circuli ſemidiametrum $= a$ intelligitur poni poſſe $\frac{a^4}{a^4+abx+x^4} = \frac{aaK+aakx}{1+2EM \times x + xx} + \frac{aaL+aalx}{1-2EO \times x + xx}$, & erit K $= L = \frac{1}{2}$, k $= \frac{1}{4EM}$, & l $= -\frac{1}{4EO}$; item poni poſſe $\frac{a^4}{a^4-abx^2+x^4} = \frac{aaK+aakx}{1-2EP \times x + xx} + \frac{aaL+aalx}{1+2EN \times x + xx}$, ita ut ſit K $= L = \frac{1}{2}$, k $= -\frac{1}{4EP}$, & l $= \frac{1}{4EN}$.

Denique ductis E G, E K, E H, E I, ope tabulæ 13 in priori tabularum ſerie in opere *Coteſii*, inveniemus aream curvæ, cujus ordinata

eſt

est $\frac{a^4}{a^4+abx^3+x^4}$, esse $=\frac{1}{4}$ in $\frac{AEq}{EM}$ (LG:GE) $+\frac{AEq}{GM}$ (LGE) $-\frac{AEq}{EO}$ (LH:HE) $+\frac{AEq}{HO}$ (LHE). Et invenietur area curvæ, cujus ordinata est $\frac{a^4}{a^4-abx^3+x^4}$, $=\frac{1}{4}$ in $\frac{AEq}{EP}$ (LI:IE) $+\frac{AEq}{IP}$ (LIE) $+\frac{AEq}{EN}$ (LK:KE) $+\frac{AEq}{KN}$ (LKE).

Jam vero his explicatis curvam, cujus ordinata est $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$,

facile dimetiemur. Est enim illa ordinata $=\frac{dz^{\frac{1}{2}n-1}}{e}\times\frac{\frac{e}{g}}{\frac{e}{g}\pm\frac{f}{g}z^n+z^{2n}}$. Et si ponamus quantitatem $\frac{\frac{e}{g}}{\frac{e}{g}\pm\frac{f}{g}z^n+z^{2n}}=\frac{a^4}{a^4\pm abx^3+x^4}$, erit $a=\sqrt[4]{\frac{e}{g}}$, $ab=\frac{f}{g}$, & $b=\frac{f}{\sqrt[4]{eg^3}}$,

item erit $x = z^{\frac{1}{n}n}$, unde erit area curvæ, cujus ordinata est $\frac{dz^{\frac{2}{n}n-1}}{e} \times$

$\frac{e}{g}$

$\frac{e}{g} \pm \frac{f}{g} z^n + z^{2n}$ = areæ curvæ cujus ordinata est $\frac{2d}{ne} \times \frac{a^4}{a^4 \pm abx^3 + x^4}$.

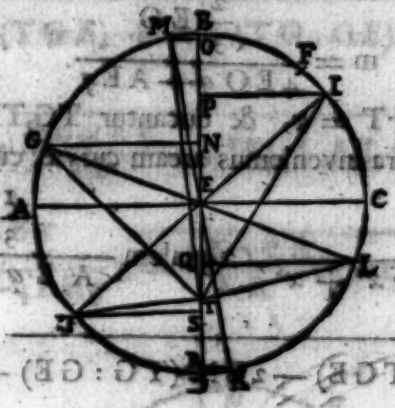
Ideoque area curvæ, cujus ordinata est $\frac{dz^{\frac{1}{2}n-1}}{e \pm fz^n + gz^{2n}}$, quando habetur $+f$, erit $= \frac{d}{2ne}$ in $\frac{AEq}{EM}$ (LG : GE) $+ \frac{AEq}{GM}$ (LGE) $- \frac{AEq}{EO}$ (LH : HE) $+ \frac{AEq}{HO}$ (LHE); quando autem habetur $-f$, erit illa area $= \frac{d}{2ne}$ in $- \frac{AEq}{EP}$ (LI : IE) $+ \frac{AEq}{IP}$ (LIE) $+ \frac{AEq}{EN}$ (LK : KE) $+ \frac{AEq}{KN}$ (LKE).

Jam hac area inventa dantur omnium curvarum areæ, quarum ordinatæ sunt $\frac{dz^{\theta n + \frac{1}{2}n - 1}}{e \pm fz^n + gz^{2n}}$, cuicunque numero integro sive affirmativo sive negativo θ sit æqualis. Nam ordinata $\frac{dz^{\frac{1}{2}n-1}}{e \pm fz^n + gz^{2n}}$ æqualis est $\frac{dz^{-\frac{1}{2}n-1}}{g \pm fz^{-n} + ez^{-2n}}$; ideoque metiemur hanc curvam, eodem modo quo eam, quam tractavimus. Datis autem duarum ex serie curvarum hic memoratarum areis, dantur areæ omnium.

Porro metiendæ sint curvæ, quarum ordinatæ sunt $\frac{dz^{\frac{1}{2}n-1}}{e \pm fz^n + gz^{2n}}$ & $\frac{dz^{\frac{3}{2}n-1}}{e \pm fz^n + gz^{2n}}$. Si describatur circulus ABCD, cujus diame-

tri

tri ſibi mutuo normales ſint A E C, B E D ; & ſi in hoc circulo ſumatur arcus quilibet A F, cujus pars ſexta ſit A G, & arcus A H ſit pars



ſexta arcûs A C F ; deinde ſi dividatur circulus totus in tres partes æquales in G, I, K, item in totidem partes æquales in H, L, M; & denique demittantur in B D perpendicula G N, M O, I P, L Q, K R, H S: His poſitis, ſi ſumatur ſemidiameter A E = a, chorda arcus A F = b, erit $\frac{a^4}{a^4 - a^2 b x^2 + x^4} = \frac{aaK + aakx}{aa + 2EN \times x + xx} +$

$\frac{aaL + aal}{aa + 2EP \times x + xx} + \frac{aaM + aam}{aa - 2ER \times x + xx}$; ubi eſt $K =$

$\frac{1}{3} = L = M$; $k = \frac{\frac{2}{3}EN}{4EN^2 - AE^2}$, $l = \frac{\frac{2}{3}EP}{4EP^2 - AE^2}$,

$m = -\frac{\frac{2}{3}ER}{4ER^2 - AE^2}$; & erit $\frac{a^4}{a^4 + aab x^2 + x^4} =$

$\frac{aaK + aak}{aa - 2ES \times x + xx} + \frac{aaL + aal}{aa - 2EQ \times x + xx} + \frac{aaM + aam}{aa + 2EO \times x + xx}$

poſitis

positis $K = L = M = \frac{1}{3}$, $k = -\frac{\frac{2}{3}ES}{4ESq - AEq}$, $l = -\frac{\frac{2}{3}EQ}{4EQq - AEq}$, $m = \frac{\frac{2}{3}EO}{4EOq - AEq}$.

Jam si sumatur $ET = x$, & ducantur TG, TM, TI, TL, TK, TH, eodem modo ut supra inveniemus aream curvæ, cujus abscissa est x & ordinata $\frac{a^4}{a^4 - aabx^3 + x^4}$, æqualem $\frac{\frac{1}{3}AEq}{AEq - 4ENq} \times \frac{AEq - 2ENq}{GN}(TGE) - 2EN(TG:GE) + \frac{\frac{1}{3}AEq}{AEq - 4EPq} \times \frac{AEq - 2EPq}{IP}(TIE) - 2EP(TI:IE) + \frac{\frac{1}{3}AEq}{AEq - 4ERq} \times \frac{AEq - 2ERq}{KR}(TKE) + 2ER(TK:KE)$. Item curvæ, cujus abscissa est x & ordinata $\frac{a^4}{a^4 + aabx^3 + x^4}$, æqualis erit $\frac{\frac{1}{3}AEq}{AEq - 4ESq} \times \frac{AEq - 2ESq}{HS}(THE) + 2ES(TH:HE) + \frac{\frac{1}{3}AEq}{AEq - 4EQq} \times \frac{AEq - 2EQq}{LQ}(TLE) + 2ES(TL:LE) + \frac{\frac{1}{3}AEq}{AEq - 4EOq} \times \frac{AEq - 2EOq}{MO}(TME) - 2EO(TM:ME)$.

Porro area curvæ, cujus abscissa est x & ordinata $\frac{a^4x}{a^4 - aabx^3 + x^4}$,

invenietur

invenietur æqualis $AEq \times kx + \frac{\frac{1}{3}AEc}{AEq - 4ENq} \times$

$\frac{AE \times EN}{GN}(TGE) + AE(TG:GE) + AEq \times lx +$

$\frac{\frac{1}{3}AEc}{AEq - 4EPq} \times \frac{AE \times EP}{IP}(TIE) + AE(TI:IE) + AEq$

$\times mx + \frac{\frac{1}{3}AEc}{AEq - 4ERq} \times -\frac{AE \times ER}{KR}(TKE) + AE(TK:KE);$

ubi erit $k = \frac{\frac{2}{3}EN}{4ENq - AEq^2}$, $l = \frac{\frac{2}{3}EP}{4EPq - AEq}$, & $m = -\frac{\frac{2}{3}ER}{4ERq - AEq}$. Hic autem convenit quantitates $AEq \times kx$, $AEq \times lx$, $AEq \times mx$ in unam summam colligere; quod efficiendum est ratione sequenti, si pro $2EN$, $2EP$, $-2ER$ promiscue ponatur m, hæc quantitas $\frac{m}{mm - aa}$ unamquamque quan-

titatum 3 k, 3 l, 3 m promiſcue denotabit: eſt autem $m^3 - 3aam + aab = 0$; ſi igitur pro $\frac{m}{mm-aa}$ ſcribatur y inveniemus $y^3 - \frac{3}{4aa-bb}y + \frac{b}{4a^4-aabb} = 0$, cujus æquationis radices erunt 3 k, 3 l, 3 m; & cum termino ſecundo indigeat, ſumma radicum nihilo erit æqualis; ideoque erit $AEq \times kx + AEq \times lx + AEq \times mx = 0$. Jam vero aream curvæ, cujus abſciſſa eſt x & ordinata $\frac{a^4x}{a^4+aabx^3+x^6}$, eodem modo inveniemus æqualem $AEq \times kx + \frac{\frac{1}{3}AEc}{AEq-4ESq} \times -\frac{\overline{AE \times ES}}{HS}(THE) + AE(TH:HE) + AEq \times lx + \frac{\frac{1}{3}AEc}{AEq-4EQq} \times -\frac{\overline{AE \times EQ}}{LQ}(TLE) + AE(TL:LE) + AEq \times mx + \frac{\frac{1}{3}AEc}{AEq-4EOq} \times -\frac{\overline{AE \times EO}}{MO}(TME) + AE(TM:ME)$; ubi eſt $k = -\frac{\frac{2}{3}ES}{4ESq-AEq}$, $l = -\frac{\frac{2}{3}EQ}{4EQq-AEq}$, & $m = \frac{\frac{2}{3}EO}{4EOq-AEq}$; & invenietur ut antea $AEq \times kx + AEq \times lx + AEq \times mx = 0$.

Jam eſt ordinata $\frac{dz^{\frac{1}{2}n-1}}{e \pm fz^n + gz^{2n}} = \frac{dz^{\frac{1}{2}n-1}}{e} \times \frac{e}{e \pm fz^n + gz^{2n}}$, & ordinata $\frac{dz^{\frac{1}{2}n-1}}{\frac{e}{g} \pm \frac{f}{g}z^n + z^{2n}} = \frac{dz^{\frac{1}{2}n-1}}{e} \times$

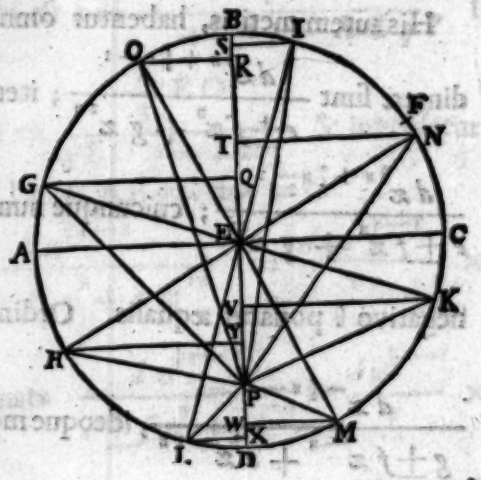
e

$\frac{\frac{e}{g}}{\frac{e}{g} \pm \frac{f}{g} z^n + z^{2n}}$. Si vero ponatur $\frac{\frac{e}{g}}{\frac{e}{g} \pm \frac{f}{g} z^n + z^{2n}} = \frac{a^4}{a^4 \pm a^2 b x + x^4}$, erit $a = \sqrt[4]{\frac{e}{g}}$, $b = \frac{f}{\sqrt{egg}}$; item est $x = z^{\frac{1}{4}n}$, ideoque area curvæ, cujus ordinata est $x \frac{dz^{\frac{1}{4}n-1}}{e \pm f z^n + g z^{2n}}$ æqualis erit areæ curvæ, cujus abscissa est x & ordinata $\frac{3d}{ne} \times \frac{a^4}{a^4 \pm a a b x^3 + x^4}$; area quoque curvæ, cujus ordinata est $\frac{dz^{\frac{3}{4}n-1}}{e \pm f z^n + g z^{2n}}$, æqualis est areæ curvæ, cujus abscissa est x & ordinata $\frac{3d}{ne} \times \frac{a^4 x}{a^4 \pm a a b x^3 + x^4}$. Hinc areas curvarum, quarum ordinatæ sunt $\frac{dz^{\frac{1}{4}n-1}}{e \pm f z^n + g z^{2n}}$ & $\frac{dz^{\frac{3}{4}n-1}}{e \pm f z^n + g z^{2n}}$, metiri possumus.

His autem metitis, habentur omnium curvarum areæ, quarum ordinatæ sunt $\frac{dz^{\theta n + \frac{1}{4}n - 1}}{e \pm f z^n + g z^{2n}}$; item omnium, quarum ordinatæ sunt $\frac{dz^{\theta n + \frac{3}{4}n - 1}}{e \pm f z^n + g z^{2n}}$; cuicunque numero integro sive affirmativo sive negativo θ ponatur æqualis. Ordinata enim $\frac{dz^{\frac{3}{4}n-1}}{e \pm f z^n + g z^{2n}} =$ est $\frac{dz^{-\frac{5}{4}n-1}}{g \pm f z^{-n} + e z^{-2n}}$, ideoque metiri possumus; data autem mensu-

ra duarum curvarum ordinatas $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$ & $\frac{dz^{\frac{3}{2}n-1}}{e\pm fz^n+gz^{2n}}$ habentium, omnes curvas, quarum ordinatæ ſunt $\frac{dz^{\theta n+\frac{1}{2}n-1}}{e\pm fz^n+gz^n}$, metiri licet. Haud difficilius metiamur curvam, cujus ordinata eſt $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$, eſt enim hæc ordinata $=\frac{dz^{-\frac{1}{2}n-1}}{g\pm fz^{-n}+ez^{-2n}}$; ſed datis areis curvarum, quarum ordinatæ ſunt $\frac{dz^{\frac{3}{2}n-1}}{e\pm fz^n+gz^{2n}}$ & $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$, dantur areæ omnium curvarum, quarum ordinatæ ſunt $\frac{dz^{\theta n+\frac{3}{2}n-1}}{e\pm fz^n+gz^{2n}}$.

Denique metiendæ ſint curvæ, quarum ordinatæ ſunt $\frac{dz^{\frac{3}{4}n-\frac{1}{2}}}{e\pm fz^n+gz^{2n}}$, & $\frac{dz^{\frac{1}{4}n-1}}{e\pm fz^n+gz^{2n}}$. Deſcribatur circulus ABCD, in quo diametri normales ſint A E C, B E D. In hoc circulo ſumpto puncto F, ſit arcus AG pars octava arcus AF, & arcus AH pars octava arcus ACF, dividatur circulus totus in partes quatuor æquales in G, I, K, L; & in H, M, N, O. Deinde ſi pro circuli radio ſcribatur *a*, & *b* pro chorda arcus CF, curvæ, quarum ordinatæ ſunt



a

$\frac{a^3}{a^3 \pm a^2 b x^2 + x^3}$ & $\frac{a^2 x^2}{a^3 \pm a^2 b x^2 + x^3}$ metiri poſſumus, ſumendo $EP = x$, & ducendo PG, PO, PI, PN, PK, PM, PL, PH, demittendoque in BD perpendicula GQ, OR, IS, NT, KV, MW, LX, HY. Si curvæ ordinata eſt $\frac{a^3}{a^3 - a^2 b x^2 + x^3}$, erit area

$$\frac{\frac{1}{16} AEq}{AEq - 2EQq} \times \frac{\overline{4GQq - AEq}}{GQ} (PGE) + \frac{AEq - 4EQq}{EQ}$$

$$(GP : GE) + \frac{\frac{1}{16} AEq}{AEq - 2ESq} \times \frac{\overline{4ISq - AEq}}{IS} (PIE) +$$

$$\frac{AEq - 4ESq}{ES} (PI : IE) + \frac{\frac{1}{16} AEq}{AEq - 2EVq} \times \frac{\overline{4KVq - AEq}}{KV}$$

$$(PKE) + \frac{4EVq - AEq}{EV} (PK : KE) + \frac{\frac{1}{16} AEq}{AEq - 2EXq}$$

$$\times \frac{\overline{4LXq - AEq}}{LX} (PLE) + \frac{4EXq - AEq}{EX} (PL : LE);$$ vel

brevius erit hæc area $= \frac{\frac{1}{16} AEq}{AEq - 2EQq} \times \frac{\overline{4GQq - AEq}}{GQ}$

$$(PGE + PKE) + \frac{AEq - 4EQq}{EQ} (PG : PK) + \frac{\frac{1}{16} AEq}{AEq - 2ESq}$$

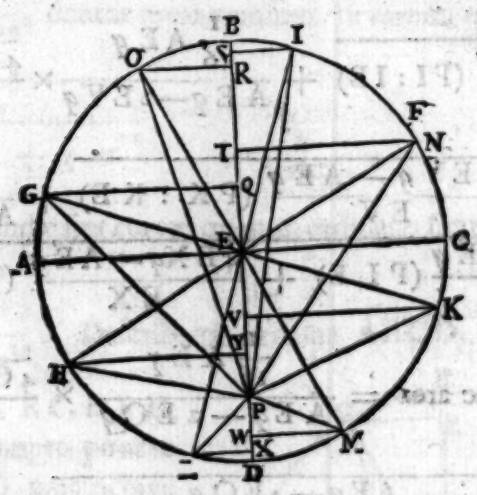
$$\times \frac{\overline{4ISq - AEq}}{IS} (PIE + PLE) + \frac{AEq - 4ESq}{ES}$$

$(PI : PL)$. Si autem curvæ ordinata ſit $\frac{a^3}{a^3 + a^2 b x^2 + x^3}$, erit

ejus area $= \frac{\frac{1}{16} AEq}{AEq - 2EYq} \times \frac{\overline{4HYq - AEq}}{HY} (PHE + PNE)$

$$+\frac{4EYq-AEq}{EY}(PH:PN)+\frac{\frac{1}{16}AEq}{AEq-2EWq}\times\frac{4MWq-AEq}{MW}(PME+POE)+\frac{4EWq-AEq}{EW}(PM:PO).$$

Porro si ordinata curvæ sit $\frac{a^3x^3}{a^3-a^3bx^4+x^7}$, erit ejus area $=\frac{1}{3}AEq\times k\,xx+\frac{1}{3}-4EQ\times k\times x+m\,xx+\frac{1}{3}+4EV\times m\times x-\frac{\frac{1}{16}AEqq}{AEq-2EQq}\times\frac{AEq}{GQ}(PGE+PKE)$



$$+\frac{AEq}{EQ}(PG:PK)+\frac{1}{3}AEq\times k\,x^3+\frac{1}{3}-4ES\times l\times x+n\,xx+\frac{1}{3}+4EX\times n\times x-\frac{\frac{1}{16}AEqq}{AEq-2ESq}\times\frac{AEq}{IS}(PIE+PLE)+\frac{AEq}{ES}(PI:PL);$$ ubi est $k=\frac{AEq-4EQq}{16EQ\times AEq-2EQq^2}$

$l=$

$l = \frac{AEq - 4ESq}{16ES \times AEq - 2ESq}$, $m = -\frac{AEq - 4EVq}{16EV \times AEq - 2EVq}$, $n = -\frac{AEq - 4EXq}{16EX \times AEq - 2EXq}$. Hic autem, cum EV sit = EQ, & EX = ES, erit $k + m = 0$, & $l + n = 0$; etiam $\frac{1}{4} - 4EQ \times k + \frac{1}{4} + 4EV \times m = 1 - \frac{AEq - 4EQq}{2AEq - 4EQq} = \frac{AEq}{2AEq - 4EQq}$, item $\frac{1}{4} - 4ES \times l + \frac{1}{4} + 4EX \times n = \frac{AEq}{2AEq - 4ESq}$; si igitur pro 2 EQ, vel 2 ES promiscue scribatur m, utraque harum quantitatum promiscue denotabitur per $\frac{aa}{2aa - mm}$; si autem ponamus $\frac{aa}{2aa - mm} = y$, inveniemus $yy - \frac{a}{b + 2a} = 0$; propterea quod ob circuli divisionem est $m^4 - 4aa\,mm + 2a^4 - a^3 b = 0$; æquatio autem $yy - \frac{a}{b + 2a} = 0$, radices duas habet æquales sub signis contrariis: quantitas igitur $\frac{aa}{2aa - mm}$ duos habet valores æquales sub signis contrariis: ideoque duæ quantitates $\frac{1}{4} - 4EQ \times k + \frac{1}{4} + 4EV \times m$, & $\frac{1}{4} - 4ES \times l + \frac{1}{4} + 4EX \times n$, simul additæ nihil efficiunt, & area curvæ, cujus ordinata est $\frac{a^3 x^3}{a^4 - a^3 b x^4 + x^8}$, erit $= -\frac{\frac{1}{16} AEqq}{AEq - 2EQq} \times \frac{AEq}{GQ}(PGE + PKE) + \frac{AEq}{EQ}(PG : PK) - \frac{\frac{1}{16} AEqq}{AEq - 2ESq} \times \frac{AEq}{IS}(PIE + PLE) + \frac{AEq}{ES}(PI : IL)$. Eodem modo si ordinata curvæ sit $\frac{a^3 x^3}{a^4 + a^3 b x^4 + x^8}$, erit ejus area $= -$

$$\frac{\frac{1}{16}AEqq}{AEq-2EYq}\times\frac{AEq}{HY}(PHE+PNE)-\frac{AEq}{EY}$$

$$\overline{(PH:PN)}-\frac{\frac{1}{16}AEqq}{AEq-2EWq}\times\frac{AEq}{MW}(PME+POE)$$

$$\overline{-\frac{AEq}{EW}(PM:PO)}.$$

Jam est ordinata $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^{n}+gz^{2n}}=\frac{dz^{\frac{1}{2}n-1}}{e}\times$

$$\frac{\frac{e}{g}}{\frac{e}{g}\pm\frac{f}{g}z^{n}+z^{2n}},\ \&\ \text{ordinata}\ \frac{dz^{\frac{1}{2}n-1}}{e\pm fz^{n}+gz^{2n}}=\frac{dz^{\frac{1}{2}n-1}}{e}\times$$

$$\frac{\frac{e}{g}}{\frac{e}{g}\pm\frac{f}{g}z^{n}+z^{2n}}.\ \text{Si vero ponatur}\ \frac{\frac{e}{g}}{\frac{e}{g}\pm\frac{f}{g}z^{n}+z^{2n}}=$$

$$\frac{a^{2}}{a^{2}\pm a^{2}bx^{2}+x^{4}},\ \text{erit}\ a=\sqrt[4]{\frac{e}{g}},\ b=\frac{f}{\sqrt{e^{1}g^{1}}};\ \text{item erit}\ z$$

$=x^{\frac{2}{n}}$, ideoque area curvæ; cujus ordinata est $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^{n}+gz^{2n}}$, æqualis erit areæ curvæ, cujus abscissa est x & ordinata $\frac{4d}{ne}\times$

$$\frac{a^{2}}{a^{2}\pm a^{2}bx^{2}+x^{4}};\ \text{etiam area curvæ, cujus ordinata est}$$

$$\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^{n}+gz^{2n}},\ \text{æqualis erit areæ curvæ, cujus abscissa est}\ x\ \&$$

ordinata $\frac{4d}{ne}\times\frac{a^{n}x^{2}}{a^{2}\pm a^{2}bx^{2}+x^{4}}$. Hinc areas curvarum, quarum

ordinatæ

ordinatæ sunt $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$ & $\frac{dz^{\frac{1}{4}n-1}}{e\pm fz^n+gz^{2n}}$, metiri licet.

Et his metitis, metiri quoque possumus omnes curvas, quarum ordinatæ sunt $\frac{dz^{\theta n+\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$, & $\frac{dz^{\theta n+\frac{1}{4}n-1}}{e\pm fz^n+gz^{2n}}$; cuicunque numero integro θ ponatur æqualis, reducendo ordinatam $\frac{dz^{\frac{1}{2}n-1}}{e\pm fz^n+gz^{2n}}$ ad $\frac{dz^{-\frac{1}{2}n-1}}{g\pm fz^{-n}+ez^{-2n}}$ & ordinatam $\frac{dz^{\frac{1}{4}n-1}}{e\pm fz^n+gz^{2n}}$ ad $\frac{dz^{-\frac{1}{4}n-1}}{g\pm fz^{-n}+ez^{-2n}}$.

Jam his computationibus tabulas 30, 33, 34 in serie posteriori tabularum *Cotesii* ad nostras has rationes reduxi, & curvas tabularum 31, 32 dimensus sum iis in casibus, qui a *Cotesio* prætermittuntur.

FINIS.

ERRATA.

IN Epistola, pag. 1. lin. 18. leg. Quoniam hæc.

Pag. 37. lin. 16. leg. $q=\frac{g}{2rs}$. lin. 17. leg. $\frac{gg}{2rs}$. lin. 18. leg. $\frac{gg}{2rs}$.

In Appendice, pag. 1. lin. ult. leg. ut 1 ad 2λ.

Pag. 18. lin. 11. leg. 2EQ (TL : LE).

Pag. 17 & 19. duc lineam TK.

Pag. 20. lin. 10. dele — ante $\frac{AE\times EO}{MO}$.

Pag. 21. lin. 3. dele x.

Catalogus Librorum profantium apud GUL. & JOH. INNYS.

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II. Opticæ: ſive de Reflectionibus, Refractionibus, Inflexionibus & Coloribus Lucis, Libri tres. Authore *Iſaaco Newtono*, Equite Aurato. Editio ſecunda auctior. 8vo. *Lond.* 1719.

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IV. Analyſis per quantitatum Series, Fluxiones, ac Differentias cum Enumeratione Linearum tertii Ordinis, 4to. 1711.

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